Jonckheere-Terpstra test

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STAT 6500

Jonckheere-Terpstra test
(Jonckheere trend test)

- Nonparametric test
- Similar to the Kruskal-Wallis test
- More statistical power in ordered alternative

Hypothesis

- H_0: $\theta_1=\theta_2=\theta_3=\ldots=\theta_k$
- H_A: $\theta_1\leq\theta_2\leq\theta_3\leq\ldots\leq\theta_k$ (with at least one strict inequality)
- $k>2$
- Θ_i is the population median for *i*th population

$$Z = \frac{U - E(U)}{\sqrt{Var(U)}}$$

For large N and the individial n_i not too small, the distribution of test statistic Z is approximately standard normal.

Where

U_{xy} is the number of observations in group y that are greater than each observation in group x.

$$E(U) = \frac{N^2 - \sum_i n_i^2}{4}$$

N: total sample size

n_i: sample number in each group

$$Var(U) = \frac{N^2(2N+3) - \sum_i [n_i^2(2n_i+3)]}{72}$$

Examples

- Hinkley (1989) gives braking distances taken by motorists to stop when travelling at various speeds. A subset of his data is shown as follow.

Speed (mph)	Braking distances (feet)			
20	48			
25	33	59	48	56
30	60	101	67	
35	85	107		

Hypothesis

- H_0: braking distance is independent of initial speed.
- H_A: braking distance increases when initial speed increases.

- Isn't that obvious?

Test statistic calculation

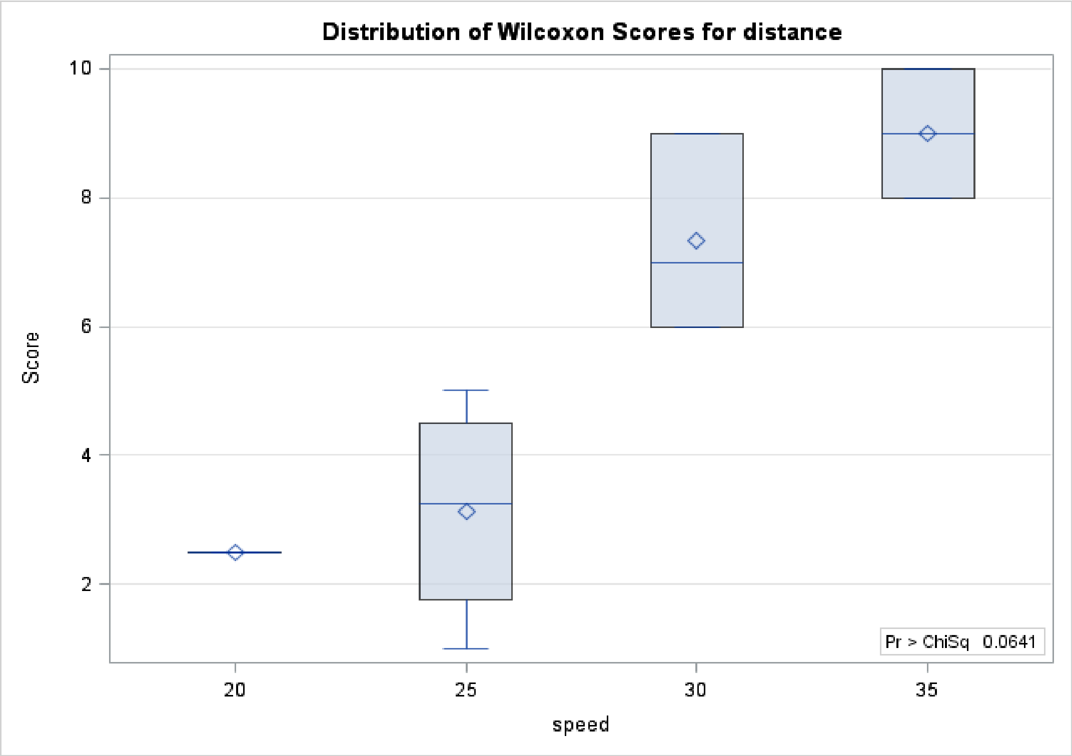
- $U_{xy} = U_{12}+U_{13}+U_{14}+U_{23}+U_{24}+U_{34} = 2.5+3+2+12+8+5=32.5$
- $E(U) = 17.5$
- $Var(U) = 27.9$
- $Z = 2.84$
- One-tail $p=0.0022$

SAS code

```
data breaking;
input speed distance @@;
cards;
20 48
25 33
25 59
25 48
25 56
30 60
30 101
30 67
35 85
35 107
;
proc npar1way wilcoxon data=breaking;
class speed;
var distance;
title1 'Kruskal-Wallis Test';
run;
proc freq;
table speed*distance / jt nopercent nocol norow;
run;
```

Outputs

Not significant result in Kruskal-Wallis Test. However, significant result in Jonckheere-Terpstra Test.



Kruskal-Wallis Test

Chi-Square	7.2599
DF	3
Pr > Chi-Square	0.0641

Jonckheere-Terpstra Test

Statistic	32.5000
Z	2.8489
One-sided Pr > Z	0.0022
Two-sided Pr > |Z|	0.0044

R code

```
> install.packages("clinfun")
> library(clinfun)
> speed20<-c(48)
> speed25<-c(33,59,48,56)
> speed30<-c(60,101,67)
> speed35<-c(85,107)
> speed<-c(speed20,speed25,speed30,speed35)
> distance<-c(rep(1,1),rep(2,4),rep(3,3),rep(4,2))
> result<-kruskal.test(speed,distance)
> result
> pieces<-list(speed20, speed25, speed30, speed35)
> n<-c(1,4,3,2)
> grp<-as.ordered(factor(rep(1:length(n),n)))
>jonckheere.test(unlist(pieces),grp,alternative="increasing")
```

R output

Kruskal-Wallis rank sum test
data: speed and distance
Kruskal-Wallis chi-squared = 7.2599, df = 3, p-value = 0.06406

Jonckheere-Terpstra test
data:
JT = 32.5, p-value = 0.002263
alternative hypothesis: increasing

Conclusion

- There is clear evidence that increasing speed increases braking distance.
- When the speed and braking distance are revelant, the Jonckheere-Terpstra test is generally more powerful than Kruskal-Wallis test (higher probability that the test will reject the H_0 when the H_A is true).

THANK YOU