Math 1100 Final Name: Solutions

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Directions:Work all problems included. If you need more room use the back of the page to complete the problem. You may a calculator for computing numerical values. However, you may not use calculators for symbolic calculations unless the problem indicates this can be done. For full credit you must show all work including algebraic steps.

Problem 1: Is the following function continuous on the interval $[-1, 0]$.

$$f(x) = \frac{x+2}{x}$$

Explain your answer. If necessary, use one-sided limits at the endpoints to justify your answer.

There may be a problem at $x=0$. So

$$\lim_{x \to 0^-} \frac{x+2}{x}$$

Since $\lim_{x \to 0^-} (x+2) = 2$ and $\lim_{x \to 0^-} x = 0$, the limit in this problem does not exist. So, f is not continuous on $[-1, 0]$ since $x=0$ is included

Problem 2: Suppose that the you know the following limits at $x = 4$

$$\lim_{x \to 4} f(x) = 2, \quad \lim_{x \to 4} g(x) = \frac{3}{2}, \quad \text{and } \lim_{x \to 4} h(x) = -1,$$

compute the following limits.

$\lim_{x \to 4}[h(x) - 3f(x)] =$ $-1 - 3(2) = -7$

$\lim_{x \to 4}(-2 - h(x)) =$ $-2 - (-1) = -2+1 = -1$

$\lim_{x \to 4} 2h(x)g(x) =$ $2(-1)(3/2) = -3$

Problem 3: Decide whether the variables in the differential equation can be separated.

$$x\frac{dy}{dx} = \frac{x^2}{y^2}$$

$$\Rightarrow x \frac{dy}{dx} dx = \frac{x^2}{y^2} dx$$

$$\Rightarrow x\, dy = \frac{x^2}{y^2} dx$$

$$\Rightarrow y^2\, dy = \frac{x^2}{x} dx \Rightarrow \underbrace{y^2}_{\text{depends only on } y} dy = \underbrace{x}_{\text{depends only on } x} dx \Rightarrow \text{yes}$$

Problem 4: Compute the following antiderivative.

$$\int 7x^{-2}dx$$

$$= 7\int x^{-2} dx$$

$$= 7 \frac{1}{-1} x^{-1} + C$$

$$= -7x^{-1} + C$$

$$= -7/x + C$$

Problem 5: Compute the derivative of the following function using the quotient rule.

$$f(x) = \frac{x-2}{7-x}$$

$$f'(x) = \frac{\left(\frac{d}{dx}(x-2)\right)(7-x) - (x-2)\frac{d}{dx}(7-x)}{(7-x)^2}$$

$$= \frac{(1)(7-x) - (x-2)(-1)}{(7-x)^2}$$

$$= \frac{(7-x)+(x-2)}{(7-x)^2} = \frac{7-x+x-2}{(7-x)^2}$$

$$= 5/(7-x)^2$$

Problem 6: Find the absolute extrema of the function on the closed interval

$$f(x) = 5 - 2x^2 \qquad [0, 3]$$

1) $f'(x) = -4x = 0$

2) evaluat f at the critical pts.

$f(0) = 5 - 2(0)^2 = 5$

same

3) evaluate at the endpts

$f(0) = 5$

$f(3) = 5 - 2(3)^2$

$= 5 - 2(9)$

$= 5 - 18 = -13$

f has an absolute minimum at $x=3$ and $f(3) = -13$

f has an absolute maximum at $x=0$ and $f(0) = 5$

Problem 7: Use integration by parts to compute the following defintie integral.

$$\int_0^1 tln(2t+1)\ dt$$

$u = \ln(2t+1)$ $\quad dv = t\,dt$

$du = \frac{1}{2t+1} \cdot (2)$ $\quad v = \frac{1}{2}t^2$

$= \frac{2}{2t+1}$

$$\int_0^1 t \ln(2t+1)\,dt = \frac{1}{2}t^2 \ln(2t+1)\Big|_0^1 - \int_0^1 \frac{1}{2}t^2 \cdot \frac{2}{2t+1}\,dt$$

$$= \left(\frac{1}{2}(1)^2 \ln(3) - 0\right) - \int_0^1 \frac{t^2}{2t+1}\,dt \longrightarrow \text{over} \longrightarrow$$

Problem 8: Find the second partial derivatives of

$$z = x^2y^3 - 2xy + 3y^2 - x^2 - 40$$

That is, compute

$$\frac{\partial^2 z}{\partial x^2}, \quad \frac{\partial^2 z}{\partial x \partial y}, \quad \frac{\partial^2 z}{\partial y \partial x}, \quad \frac{\partial^2 z}{\partial y^2}$$

$\frac{\partial z}{\partial x} = 2xy^3 - 2y - 2x$

$\frac{\partial z}{\partial y} = 3x^2y^2 - 2x + 6y$

$\frac{\partial^2 z}{\partial x^2} = 2y^3 - 2$

$\frac{\partial^2 z}{\partial x \partial y} = 6xy^2 - 2$

$\frac{\partial^2 z}{\partial y \partial x} = 6xy^2 - 2$

equal

$\frac{\partial^2 z}{\partial y^2} = 6x^2y + 6$

$$= \frac{1}{2}\ln(3) - \int_0^1 \frac{t^2}{2t+1}\,dt$$

$u = 2t+1 \Rightarrow t=0, u=1 \rightarrow t = \frac{1}{2}(u-1)$

$du = 2\,dt \Rightarrow t=1, u=3 \Rightarrow t^2 = \frac{1}{4}(u-1)^2$

$$= \ln(\sqrt{3}) - \int_1^3 \frac{\frac{1}{4}(u-1)^2}{u} \cdot \frac{1}{2}\,du$$

$$= \ln(\sqrt{3}) - \frac{1}{8}\int_1^3 \frac{u^2-2u+1}{u}\,du$$

$$= \ln(\sqrt{3}) - \frac{1}{8}\int_1^3 \left(u - 2 + \frac{1}{u}\right) du$$

$$= \ln(\sqrt{3}) - \frac{1}{8}\left(\frac{1}{2}u^2 - 2u + \ln|u|\right)\Big|_{u=1}^{u=3}$$

$$= \ln(\sqrt{3}) - \frac{1}{8}\left\{\left(\frac{1}{2}(3^2) - 2(3) + \ln(3)\right) - \frac{1}{2}(1)^2 - 2(1) + \ln(1)\,)\right\}$$

This is good enough.

Problem 9: Suppose we want to analyze the function $f(x) = 3x^4 - 8x^3 + 6x^2$. **a.** Determine all points where the concavity of the function is zero. **b.** Which of these points are inflection points.?

$f'(x) = 12x^3 - 24x^2 + 12x$

$f''(x) = 36x^2 - 48x + 12$

$= 12(3x^2 - 4x + 1) = 0$

$\Rightarrow \quad x = \dfrac{4 \pm \sqrt{16 - 12}}{6}$ (Quadratic Formula)

$= \dfrac{4 \pm \sqrt{4}}{6} = \dfrac{4 \pm 2}{6}$

$x_1 = 1, \; x_2 = 1/3$

$f''(0) = 12 > 0$ | $f''(\frac{2}{3}) = -4 < 0$ | $f''(2) > 0$

concave up | $1/3$ | concave dn | 1 | concave up

Since there is a change in the concavity at both $x = 1/3$, and $x = 1$ they are both inflection pts.

Problem 10: Find the equation of a tangent line to the function at the indicated point.

$g(x) = 2 + e^{3x^2} \qquad (0, 3)$

$g'(x) = 0 + e^{3x^2}(6x)$

$= 6x e^{3x^2}$

$g'(0) = 6(0) \cdot e^0 = 0$

$$\Rightarrow \begin{cases} y - y_0 = m(x - x_0) \\ m = g'(0) = 0 \\ x_0 = 0 \\ y_0 = 3 \end{cases}$$

$\Rightarrow y - 3 = 0 \cdot (x - 0)$

$\Rightarrow y - 3 = 0$

$\Rightarrow \underline{\underline{y = 3}}$