Math 1100 Test 1 Name: Solutions

Created: Sat Dec 31 13:36:50 MST 2005

Directions:Work all problems included. If you need more room use the back of the page to complete the problem. You may a calculator for computing numerical values. However, you may not use calculators for symbolic calculations unless the problem indicates this can be done. For full credit you must show all work including algebraic steps.

Problem 1: Compute the limit of the polynomial shown below at $x = -2$.

$$\lim_{x \to -2} x^3 - 4x + 1$$

Since we are evaluating the limit on a polynomial we can evaluat the limit at a point

$$\lim_{x \to -2} (x^3 - 4x + 1) = (2)^3 - 4(2) + 1 = 8 - 8 + 1 = 1$$

Problem 2: a. State the domain of the following function.

$$f(x) = x^3 - 2x^2 - 4x + 9$$

b. Is the function continuous at all points in the domain? Explain your answer.

a. The domain is all real numbers or $(-\infty, +\infty)$

b. Since $f(x)$ is a polynomial the function is continuous on its entire domain.

Problem 3: Write the definition of the derivative of a function f at a point, x. Use the definition of the derivative to compute the derivative of the following function.

$$f(x) = 3x - 7$$

Def:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Using the def:

$$f'(x) = \lim_{\Delta x \to 0} \frac{(3(x+\Delta x) - 7) - (3x - 7)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3x} + 3\Delta x - \cancel{7} - \cancel{3x} + \cancel{7}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 3 = 3$$

$$\Rightarrow f'(x) = 3$$

Problem 4: The revenue as a function of time is modeled by the function

$$R(t) = -165.2t^2 + 387.5t - 420.1$$

Compute the rate of change of the revenue at any time for this model function and determine the predicted rate of change of revenue at $t = 1$ year and $t = 2$ years.

The rate of change is:

$$\frac{dR}{dt} = \text{marginal revenue}$$

$$= -165.2(2t) + 387.5$$

$$= -330.4t + 387.5$$

For $t = 1$

$$\frac{dR}{dt} = -330.4(1) + 387.5 = 57.1$$

For $t = 2$

$$\frac{dR}{dt} = -330.4(2) + 387.5 = -273.3$$

Problem 5: For the function

$$y(x) = x^2 - 4.1x + 1.7$$

find the instantaneous rate of change of the function at $x = 2$. Compare the instantaneous rate of change to the average rate of change on the interval $[1.9, 2.0]$.

$y'(x) = 2x - 4.1$

At $x = 2$

$\Rightarrow y'(2) = 2(2) - 4.1$
$= 4 - 4.1$
$= -0.1$

$\Delta y = y(2) - y(1.9)$
$= [(2)^2 - 4.1(2) + 1.7] - [(1.9)^2 - 4.1(1.9) + 1.7]$
$= -2.5 + 2.48 = -0.02$

$\Delta x = 2 - 1.9 = 0.1$

$\frac{\Delta y}{\Delta x} = \frac{-0.02}{0.1} = -0.2$

$y'(x)$ is $\frac{1}{2}$ of the average rate of change

Problem 6: Compute the derivative of the following function using the product rule.

$$f(x) = 3(x-2)^2(7-x)^7$$

$f'(x) = 3(2)(x-2)^1(7-x)^7 + 3(x-2)^2(7)(7-x)^6(-1)$
$= 6(x-2)(7-x)^7 - 21(x-2)^2(7-x)^6$

This is far enough!

$= 3(x-2)(7-x)^6\left(2(7-x) - 7(x-2)\right)$
$= 3(x-2)(7-x)^6(14 - 2x - 7x + 14)$
$= 3(x-2)(7-x)^6(28 - 9x)$

This would be needed if you are asked to simplify

Problem 7: Compute the derivative of the following function using the chain rule.

$$f(x) = (x^3 - x^2 + 1)^2$$

$$f'(x) = 2(x^3 - x^2 + 1)^1 \cdot \frac{d}{dx}(x^3 - x^2 + 1)$$

$$= 2(x^3 - x^2 + 1)(3x^2 - 2x) \quad \leftarrow \text{far enough for this problem}$$

$$= 2x(x^3 - x^2 + 1)(3x - 2)$$

Problem 8: Given the equation

$$xy + x^2 + y^2 = 1$$

compute $\frac{dy}{dx}$ using implicit differentiation. Compute the slope of the tangent line to the graph of the solution set of the equation at the point $(1, 0)$. Write the equation of the tangent line at this point.

$$\frac{d}{dx}(xy + x^2 + y^2) = \frac{d}{dx}(1)$$

$$\Rightarrow y + xy' + 2x + 2yy' = 0$$

$$\Rightarrow y + 2x + (x + 2y)y' = 0$$

$$\Rightarrow y' = -\frac{y + 2x}{x + 2y}$$

The slope is:

$$y' = -\frac{0 + 2(1)}{1 + 2\cdot(0)} = -\frac{2}{1}$$

$$= -2$$

In point slope form

$$y - y_0 = m(x - x_0)$$

$$y - 0 = -2(x - 1)$$

$$\Rightarrow y = -2x + 2$$

Problem 9: A cube of ice is melting at a constant rate of 2 cubic inches per minute. Write a formula for the volume of a cube with edge dimension x. Determine the rate of change of the edge dimension x when the cube has a volume of 120 cubic inches.

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$-2 = 3(4.93)^2 \frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} \approx \frac{-2}{3(4.93)^2}$

≈ -0.027 in/sec.

$\frac{dV}{dt}$ = rate of change of volume
= -2 cubic inches per min.
↑ decreasing

$V = 120 \Rightarrow x = \sqrt[3]{120} \approx 4.93$

Problem 10: Determine the location of the asymptotes of the following function.

$$f(x) = \frac{x-1}{x^2 - 5x + 6}$$

Give the largest domain possible for the function. Where is this function continuous?

$f(x) = \frac{x-1}{(x-2)(x-3)}$

The vertical asymptotes are at $x=2$, $x=3$.

The largest domain is
$(-\infty, 2) \cup (2, 3) \cup (3, +\infty)$

The function is continuous on its domain and discontinuous a $x=2$, and $x=3$.