Math 1100 Test 1 Name: Solutions

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**Directions:Work all problems included. If you need more room use the back of the page to complete the problem. You may a calculator for computing numerical values. However, you may not use calculators for symbolic calculations unless the problem indicates this can be done. For full credit you must show all work including algebraic steps.**

**Problem 1:** Compute the limit of the polynomial shown below at $x = 1$.

$$\lim_{x \to 1} x^2 - 2x + 1$$

Since the function is a polynomial we only need to evaluate at the point. That is,

$$\lim_{x \to 1} x^2 - 2x + 1 = (1)^2 - 2(1) + 1 = 0$$

**Problem 2: a.** State the domain of the following function.

$$g(x) = \frac{x^2 - 4x + 3}{x - 1}$$

**b.** Is the function continuous at all points in the domain? Explain your answer.

a. First, the function is defined for all real numbers except $x = 1$ $\Rightarrow$ Domain $= (-\infty, 1) \cup (1, \infty)$

b. The function is a rational function of polynomials and so is continuous on its domain.

**Problem 3:** Write the definition of the derivative of a function $f$ at a point, $x$. Use the definition of the derivative to compute the derivative of the following function.

$$f(x) = x^2 + 3$$

Def:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

For the example,

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{((x+\Delta x)^2+3) - (x^2+3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 3 - x^2 - 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x)$$

$$= 2x.$$

**Problem 4:** Compute the derivative of the following function

$$f(x) = 4x^3 - 2x^2 + x - 1$$

Evaluate the derivative at the point $x = 1$. Then compute the equation of the tangent line to the graph of the curve at $x = 1$.

$$f'(x) = 12x^2 - 4x + 1$$

$$f'(1) = 12(1)^2 - 4(1) + 1$$

$$= 12 - 4 + 1 = 9$$

The equation of the tangent line is

$$y - y_0 = m(x - x_0)$$

$$x_0 = 1 \Rightarrow y_0 = f(1)$$

$$= 4(1)^3 - 2(1)^2 + (1) - 1$$

$$= 4 - 2 + 1 - 1 = 2$$

So,

$$y - 2 = 9(x - 1)$$

$$\Rightarrow y = 9x - 9 + 2 = 9x - 7$$

**Problem 5:** The cost function for producing a number of items is defined by

$$C(x) = 5000 + 20x$$

and the function is given by

$$R(x) = 1000x$$

Write Compute the marginal cost, revenue, and function from this cost function.

$P(x) = \text{profit} = R(x) - C(x) = 1000x - 5000 - 20x$

$= 980x - 5000$

$C'(x) = 20$

$R'(x) = 1000$

$P'(x) = 980$

**Problem 6:** Compute the derivative of the following function using the quotient rule.

$$h(x) = \frac{5x+3}{2x^2+3}$$

$$h'(x) = \frac{\left(\frac{d}{dx}(5x+3)\right)(2x^2+3) - (5x+3)\frac{d}{dx}(2x^2+3)}{(2x^2+3)^2}$$

$$= \frac{(5)(2x^2+3) - (5x+3)(4x)}{(2x^2+3)^2}$$

← this is far enough

$$= \frac{10x^2+15-20x^2-12x}{(2x^2+3)^2}$$

$$= \frac{-10x^2-12x+15}{(2x^2+3)^2}$$

**Problem 7:** The velocity (in feet per second) of and automobile starting from rest is modeled by

$$\frac{ds}{dt} = \frac{90t}{t+10}$$

Determine the acceleration for the velocity of the automobile from this formula.

$\frac{ds}{dt} = v(t) =$ velocity

acceleration $= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right)$

$= \frac{d}{dt}\left(\frac{90t}{t+10}\right) = \frac{(90)(t+10)-(90t)(1)}{(t+10)^2}$ (quotient rule)

$= \frac{90t+900-90t}{(t+10)^2} = \frac{900}{(t+10)^2}$

**Problem 8:** Given the equation

$$x^2 - 5xy + y^2 = -5$$

compute $\frac{dy}{dx}$ using implicit differentiation. Compute the slope of the tangent line to the graph of the solution set of the equation at the point $(3, 1)$. Write the equation of the tangent line at this point.

$\frac{d}{dx}(x^2-5xy+y^2) = \frac{d}{dx}(-5)$

$2x - 5y - 5xy' + 2yy' = 0$

$2x - 5y + (2y-5x)y' = 0$

$\Rightarrow y' = \frac{5y-2x}{2y-5x}$

at $x = 3$ and $y = 1$

$\Rightarrow y' = \frac{5(1)-2(3)}{2(1)-5(3)} = \frac{5-6}{2-15}$

$= \frac{-1}{-13} = \frac{-1}{13}$

The equation of the tangent line is

$y - y_0 = m(x - x_0)$

$y - 1 = \frac{-1}{13}(x-3)$

$\Rightarrow y = -\frac{1}{13}x + \frac{3}{13} + 1$

$= -\frac{1}{13}x + \frac{16}{13}$

**Problem 9:** The sales, $S$, for a company can be modeled by

$$S = 1275 + 45x + 0.25x^2$$

where $x$ gives the advertising costs associated with selling the product. If the rate of change in advertiseing are increasing at a rate of \$100 per week find the rate of change in sales.

First compute $\frac{dS}{dt}$

$$\frac{dS}{dt} = 0 + 45\frac{dx}{dt} + 0.5\,x\,\frac{dx}{dt}$$

for $dx/dt = 100$

$$\frac{dS}{dt} = (45)(100) + (0.5)(x)(100)$$

$$= 4500 + 50x$$

Without $x$ this is as far as we can go.

**Problem 10:** Compute the one-sided limit of the function shown below at $x = 2$.

$$\lim_{x \to 2^+} \frac{x^3 + x^2 + x}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{x(x^2 + x + 1)}{x - 2}$$

$$\lim_{x \to 2} x(x^2 + x + 1) = (2)(4 + 2 + 1) = 14$$

Since the numerator is 14 and the denominator tends to zero, the limit does not exist. Since $x \to 2^+$

$x - 2 > 0$ and

$$\lim_{x \to 2^+} \frac{\overbrace{x^3 + x^2 + x}^{\to 14}}{\underbrace{x - 2}_{\to 0}} = +\infty$$