Math 1100 Test 2 Name: Solutions

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Directions:Work all problems included. If you need more room use the back of the page to complete the problem. You may a calculator for computing numerical values. However, you may not use calculators for symbolic calculations unless the problem indicates this can be done. For full credit you must show all work including algebraic steps.

Problem 1: Suppose we want to analyze the function $f(x) = 3x^4 - 40x^3 + 198x^2 - 432x + 420$. **a.** Show that the critical points for the function are $x = 3$ and $x = 4$. Note that the critical point at $x = 3$ is repeated twice. **b.** Determine the intervals on which the function is increasing and decreasing. **c.** Use the first derivative test to determine if the critical points are locations of relative minimums, relative maximums, or neither.

$f'(x) = 12x^3 - 120x^2 + 396x - 432$

$f'(3) = 12(27) - 120(9) + (396)(3) - (432) = 0$

$f'(4) = 12(64) - 120(16) + (396)(4) - (432) = 0$

$f'(7/2) = 12(7/2)^3 - 120(7/2)^2 + 396(7/2) - 432$

$= -1.5 < 0$

$f'(5) = 12(125) - 120$

$f' < 0$ | $f' < 0$ | $f' > 0$

3 | 4

neither | rel. min.

Problem 2: Find all relative extrema of the function. Use the second derivative test when applicable.

$f(x) = x^3 - 5x^2 + 7x$

$f'(x) = 3x^2 - 10x + 7$

$= (3x-7)(x-1) = 0$

$x = 1, \ x = 7/3$

$f''(x) = 6x - 10$

$f''(1) = 6 - 10 = -4 < 0$

$\Rightarrow$ a relative max at $x = 1$

$f''(7/3) = 14 - 10 = 4 > 0$

$\Rightarrow$ a relative min at $x = 1$

Problem 3: Determine two numbers that have a product of 128 and have a minimum sum.

$x \cdot y = 128 \longrightarrow y = 128/x \Rightarrow y = \pm\sqrt{128}$

$x + y =$ minimum value

$= x + 128/x = S(x)$

So, we take $x = -\sqrt{128}$

$y = -\sqrt{128}$

$S'(x) = 1 - 128/x^2 = 0$

$S(x) = -2\sqrt{128} =$ min value

$\Rightarrow x^2 - 128 = 0$

$\Rightarrow x^2 = 128 \Rightarrow x = \pm\sqrt{128}$

Problem 4: Find the amount s of advertising that maximizes the profit P.

$$P = -2s^3 + 35s^2 - 100s + 200$$

$\frac{dP}{ds} = -6s^2 + 70s - 100$

$= -2(3s^2 - 35s + 50) = 0$

$s = \frac{35 \pm \sqrt{35^2 - 4(3)(50)}}{6} = \frac{35 \pm \sqrt{625}}{6}$

$= \frac{35 \pm 25}{6} \Rightarrow s = \frac{60}{6} = 10$ or $s = \frac{10}{6}$

$\frac{d^2P}{ds^2} = -12s + 70$

$s = 10 \Rightarrow \frac{d^2P}{ds^2} = -120 + 70 = -50 < 0$ relative max

$s = \frac{10}{6} \Rightarrow \frac{d^2P}{ds^2} = -24 + 70 = 46 > 0$ relative min

$\Rightarrow s = 10$ is the value.

Problem 5: Write the expression as a logarithm applied to a single expression.

$$\frac{1}{5}ln(x^3(x-1)) - 2ln(x-2)$$

$$= \ln\left((x^3(x-1))^{1/5}\right) - \ln(x-2)^2$$

$$= \ln\left(\frac{(x^3(x-1))^{1/5}}{(x-2)^2}\right)$$

Problem 6: Demand Function: Solve the demand function

$$x = ln\left(\frac{575}{2p}\right)$$

for p in terms of x. Use the result to find dp/dx. Then find the rate of change when $p = \$5$. What is the relationship between this derivative and dx/dp?

Find the solution

$$x = \ln\left(\frac{575}{2p}\right)$$

$$e^x = \frac{575}{2p}$$

$$2pe^x = 575$$

$$2p = 575e^{-x}$$

$$p = (287.5)e^{-x}$$

$$\frac{dp}{dx} = (287.5)(e^{-x}(-1))$$

$$= -287.5e^{-x}$$

$$p = 5 \Rightarrow x = \ln\left(\frac{575}{10}\right)$$

$$\frac{dp}{dx} = -287.5e^{-\ln\left(\frac{575}{10}\right)}$$

$$= -287.5e^{\ln\left(\frac{575}{10}\right)^{-1}}$$

$$= -287.5\left(\frac{575}{10}\right)^{-1}$$

$$= -287.5\left(\frac{10}{575}\right)$$

$$= -5$$

$$\frac{dx}{dp} = -1/5$$

$\frac{dx}{dp}$ is the reciprocal of $\frac{dp}{dx}$ at each point

Problem 7: Exponential decay: What percent of a present amount of radioactive radium will remain after 900 years? The half life of the radium in question is 1620 years. Recall that the form of the exponential model is $y = Ce^{rt}$ where y is the amount at any time.

The model is:

$$A = Ce^{rt}$$

If $A = \frac{1}{2}C$ (half life)

$$\frac{1}{2}\cancel{C} = \cancel{C}e^{r(1620)}$$

$$\Rightarrow \frac{1}{2} = e^{1620r}$$

$$\Rightarrow r = \frac{\ln(1/2)}{1620} \approx -0.0004278$$

At 900 years

$$A = Ce^{-0.0004278(900)}$$

$$\Rightarrow A = C(0.6804)$$

$$\Rightarrow \frac{A}{C} \approx 0.6804$$

or 68.04% of initial amount

Problem 8: Compute the following antiderivative.

$$\int \frac{2}{x^2}dx$$

$$= \int 2x^{-2}\,dx$$

$$= -2x^{-1} + C$$

$$= -\frac{2}{x} + C$$

Problem 9: Find the indefinite integral and check your result by differentiation.

$$\int \frac{3x}{2x^2 - 16} dx = 3\int \frac{4}{4} \cdot \frac{x}{\sqrt{2x^2-16}}\, dx = \frac{3}{4}\int \frac{4x}{\sqrt{2x^2-16}}\, dx$$

$u = 2x^2 - 16$

$du = 4x\, dx$

$$= \frac{3}{4}\int \frac{1}{u^{1/2}}\, du$$

$$= \frac{3}{4} \cdot 2\, u^{1/2} + C$$

$$= \frac{3}{2} u^{1/2} + C = \frac{3}{2}\sqrt{2x^2-16} + C$$

Check:

$$\frac{d}{dx}\left[\frac{3}{2}(2x^2-16)^{1/2} + C\right]$$

$$= \frac{3}{2}\left(\frac{1}{2}\right)(2x^2-16)^{-1/2}(4x) + 0 = \frac{3}{4} \cdot 4x\,(2x^2-16)^{-1/2} = \frac{3x}{\sqrt{2x^2-16}}$$

Problem 10: Use the Log Rule to find the indefinite integral.

$$\int \frac{4x}{x^2 + 1}\, dx = 2\int \frac{2x}{x^2+1}\, dx = 2\int \frac{du/dx}{u}\, dx$$

$u = x^2 + 1$

$du = 2x\, dx$

$$= 2 \ln|u| + C$$

$$= 2 \ln(x^2+1) + C$$