Math 1100 Test 2 Name: Solutions

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Directions:Work all problems included. If you need more room use the back of the page to complete the problem. You may a calculator for computing numerical values. However, you may not use calculators for symbolic calculations unless the problem indicates this can be done. For full credit you must show all work including algebraic steps.

Problem 1: Suppose we want to analyze the function $f(x) = x^3 - x^2 - 6x$. Determine the intervals on which the function is increasing and decreasing.

$f'(x) = 3x^2 - 2x - 6 = 0$

Find the roots:

$$x = \frac{2 \pm \sqrt{4 - 4(3)(-6)}}{6} = \frac{2 \pm \sqrt{76}}{6} = \frac{2 \pm 2\sqrt{19}}{6} = \frac{1}{3} \pm \frac{1}{3}\sqrt{19}$$

$f' > 0$ | $f' < 0$ | $f' > 0$

$\frac{1}{3} - \frac{1}{3}\sqrt{19}$ $\quad$ $\frac{1}{3} + \frac{1}{3}\sqrt{19}$

$f'(-10) = 3(100) - 2(-10) - 6 > 0 \Rightarrow$ inc

$f'(1/3) = 3(1/9) - 2(1/3) - 6 < 0 \Rightarrow$ dec

$f'(3) = 3(9) - 2(3) - 6 > 0 \Rightarrow$ inc

$\Rightarrow$ increasing on $(-\infty, \frac{1}{3} - \frac{1}{3}\sqrt{19}) \cup (\frac{1}{3} + \frac{1}{3}\sqrt{19}, \infty)$ dec. on $(\frac{1}{3} - \frac{1}{3}\sqrt{19}, \frac{1}{3} + \frac{1}{3}\sqrt{19})$

Problem 2: Suppose we want to analyze the function $f(x) = 3x^4 - 8x^3 + 6x^2$. **a.** Determine all critical points for the function. **b.** Use the first derivative test to determine if the critical points are locations of relative minimums, relative maximums, or neither.

$$f'(x) = 12x^3 - 24x^2 + 12x = 12x(x^2 - 2x + 1) = 12x(x-1)^2$$

$x = 0$, $x = 1$ are critical points

$f'(-1) = 12(-1)(-2)^2 = -24 < 0$

$f'(1/2) = 12(1/2)(\frac{1}{2})^2 > 0 \leftarrow$ inflect.

$f'(2) = 12(2)(2-1)^2 > 0$

$f' < 0$ | $f' > 0$ | $f' > 0$

0 $\quad$ 1

$\uparrow$ relative min $\quad$ $\uparrow$ neither a rel. max or min

Problem 3: Suppose we want to analyze the function $f(x) = 3x^4 - 8x^3$. **a.** Determine all points where the concavity of the function is zero. **b.** Which of these points are inflection points.?

$f' = 12x^3 - 24x^2$

$f'' = 36x^2 - 48x$

$= 12x(3x-4)$

$x = 0, \quad x = 4/3$

$f''>0 \;|\; f''<0 \;|\; f''>0$

$0 \qquad 4/3$

both are inflection pts.

$f''(-1) = 12(-1)(3(-1)-4) = 84 > 0$ concave up

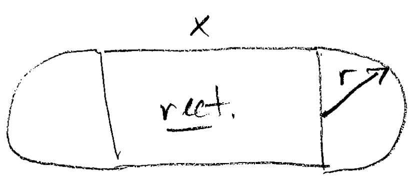
$f''(1) = 12(1)(3-4) = -12 < 0$ concave dn

$f''(2) = 12(2)(6-4) = 24 > 0$ concave up

changes

changes

Problem 4: A fitness club owner wants to maximimze the available room on the interior of a running/jogging track. The interior region of the track will be a rectangle with semicircular ends. The perimeter of the interior region should be 200 meters. What are the dimensions of the rectangle that maximizes the area of the interior of the track.



Perimeter $= 2x + 2\pi r = 200$ meters

perimeter of the circular part

$\Rightarrow x = 100 - \pi r$

The area is

$$A = x\cdot(2r) = (100 - \pi r)(2r) = 200r - 2\pi r^2$$

$$\frac{dA}{dr} = 200 - 4\pi r \Rightarrow r = \frac{200}{4\pi}$$

$$\Rightarrow x = 100 - \pi\left(\frac{200}{4\pi}\right) = 100 - \frac{200}{4} = 100 - 50 = \underline{50 \text{ meters}}$$

Check:

$$2x + 2\pi r = 2(50) + 2\pi\left(\frac{200}{4\pi}\right) = 100 + 2\pi\left(\frac{50}{\pi}\right) = 100 + 100 = 200$$

Problem 5: Maximizing Profit: The demand function for a given product is given by

$$p = \frac{50}{\sqrt{x}}$$

The cost function for this produce is given by

$$C = 0.5x + 500$$

a. Write down the function defining profit. That is, what is the function for the revenue minus the cost. **b.** Determine the price at which the profit will be maximized.

a. Profit = Revenue − Cost $\quad R = x \cdot p = x \, {}^{50}/\!\sqrt{x} = 50\sqrt{x}$

$= x \cdot p - 0.5x - 500$

$= 50\sqrt{x} - 0.5x - 500 = P(x)$

$\boxed{p = \frac{50}{\sqrt{2500}} = \frac{50}{50} = 1}$

b. $P'(x) = 25x^{-1/2} - 0.5 + 0 = 0$

$\Rightarrow 25x^{-1/2} = 1/2 \Rightarrow x^{1/2} = 50 \Rightarrow x = 2500$ = critical pt.

$P''(x) = -\frac{25}{2}x^{-3/2} \qquad P''(2500) = -\frac{25}{2}(2500)^{-3/2} < 0$

$\Rightarrow$ Local max at $x = 2500$

Problem 6: Find the equation of a tangent line to the function at the indicated point.

$$g(x) = 2 + e^{3x^2} \qquad (0, 3)$$

$g'(x) = e^{3x^2} \cdot (6x)$

$g'(0) = e^{3(0)^2} \cdot 6 \cdot 0 = 0$

$\Rightarrow y - y_0 = f'(x_0)(x - x_0)$

$\Rightarrow y - 3 = (0)(x - 0)$

$\Rightarrow y - 3 = 0 \Rightarrow \underline{y = 3}$

Problem 7: Write the expression as a sum, difference, or multiple of logarithms using properties of logarithms.

$$ln\frac{x(3x+1)}{6(2x+1)^2}$$

$= \ln(x(3x+1)) - \ln(6(2x+1)^2)$

$= \ln(x) + \ln(3x+1) - \ln(6) - \ln(2x+1)^2$

$= \ln(x) + \ln(3x+1) - \ln(6) - 2\ln(2x+1)$

Done

Problem 8: Find the derivative of the function.

$$y = ln\left(\frac{x+1}{x-2}\right)$$

Hint: Using properties of logarithms can simplify this problem.

$y = \ln(x+1) - \ln(x-2)$

$\frac{dy}{dx} = \frac{1}{x+1} - \frac{1}{x-2}$

Problem 9: Compute the following antiderivative.

$$\int 7x^{-2}dx$$

$$= \frac{7}{-1} x^{-1} + C$$

$$= -7x^{-1} + C$$

Problem 10: Find the indefinite integral and check your result by differentiation.

$$\int 3x\sqrt{1-2x^2}\,dx$$

$u = 1-2x^2$

$du = -4x\,dx$

$$\Rightarrow \int 3x\sqrt{1-2x^2}\,dx = 3\int \frac{-4}{-4} x\sqrt{1-2x^2}\,dx$$

$$= -\frac{3}{4}\int -4x\sqrt{1-2x^2}\,dx = \frac{-3}{4}\int \sqrt{1-2x^2}\,(-4x\,dx)$$

$$= -\frac{3}{4}\int \sqrt{u}\,du$$

$$= -\frac{3}{4}\cdot\frac{2}{3}u^{3/2} + C$$

$$= -\frac{1}{2}u^{3/2} + C$$

$$= -\frac{1}{2}(1-2x^2)^{3/2} + C$$