Math 1100 Test 3 Name: Solution

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Directions:Work all problems included. If you need more room use the back of the page to complete the problem. You may a calculator for computing numerical values. However, you may not use calculators for symbolic calculations unless the problem indicates this can be done. For full credit you must show all work including algebraic steps.

Problem 1: Find the first partial derivatives of the function

$$w = 4x^3y^2 - 3xyz + 7yz^2$$

$$\frac{\partial w}{\partial x} = 4(3x^2)y^2 - 3(1)yz + 0 = 12x^2y^2 - 3yz$$

$$\frac{\partial w}{\partial y} = 4x^3(2y) - 3x(1) + 7(1)z^2 = 8x^3y - 3x + 7z^2$$

$$\frac{\partial w}{\partial z} = 0 - 3xy(1) + 7y(2z) = -3xy + 14yz$$

Problem 2: Compute the following indefinite integral

$$\int_0^4 x(x+5)^4\, dx$$

$u = x+5$ $\quad x = 0 \Rightarrow u = (0+5) = 5$ $\qquad x = u - 5$

$du = dx$ $\quad x = 4 \Rightarrow u = (4+5) = 9$

$$\Rightarrow \int_0^4 x(x+5)^4 dx$$

$$= \int_5^9 (u-5)u^4\, du = \int_5^9 (u^5 - 5u^4)\, du$$

$$= \left(\frac{1}{6}u^6 - \frac{5}{5}u^5\right)\Big|_5^9 = \left(\frac{1}{6}9^6 - 9^5\right) - \left(\frac{1}{6}5^6 - 5^5\right)$$

Problem 3: Compute the general solution of the following equation via separation of variables.

$$\frac{dy}{dx} = \frac{x^2+3}{y^2-1}$$

Leave the solution in an implicit relationship for x and y.

$$\frac{dy}{dx} \cdot dx = \frac{x^2+3}{y^2-1}\, dx$$

$$\Rightarrow \quad dy = \frac{x^2+3}{y^2-1}\, dx$$

$$\Rightarrow \quad (y^2-1)\, dy = (x^2+3)\, dx$$

$$\Rightarrow \quad \int (y^2-1)\, dy = \int (x^2+3)\, dx$$

$$\frac{1}{3}y^3 - y = \frac{1}{3}x^3 + 3x + C$$

Problem 4: Use integration by parts to compute the following indefintie integral.

$$\int xe^{2x}\, dx$$

$u = x \qquad dv = e^{2x}\, dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}$

$$\Rightarrow \int xe^{2x}\, dx = \underbrace{\frac{1}{2}xe^{2x}}_{vu} - \int \underbrace{\frac{1}{2}e^{2x}}_{v}\, \underbrace{dx}_{du}$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\, dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

Problem 5: Determine whether the function

$$y = e^{3t}$$

is a solution for the following differential equation

$$y^{(3)} - 27y = 0$$

$y' = 3e^{3t}$

$y'' = y^{(2)} = 9e^{3t}$

$y^{(3)} = 27e^{3t}$

$\Rightarrow y^{(3)} - 27y = 27e^{3t} - 27(e^{3t}) = 0$

This means that $y = e^{3t}$ is a solution for the differential equation

Problem 6: Determine whether or not the improper integral converges. If it does, evaluate the integral.

$$\int_2^\infty \frac{2x}{\sqrt{3x^2-4}}\,dx$$

$= \lim_{b\to\infty} \int_2^b \frac{2x}{\sqrt{3x^2-4}}\,dx = \lim_{b\to\infty} \frac{2}{3}\sqrt{3x^2-4}\,\Big|_2^\infty$

$= \lim_{b\to\infty} \left(\frac{2}{3}\sqrt{3b^2-4} - \frac{2}{3}\sqrt{8}\right)$

this diverges as $b \to +\infty$

$\Rightarrow$ The improper integral diverges

Antiderivative:

$\int \frac{2x}{\sqrt{3x^2-4}}\,dx = \frac{1}{3}\int \frac{6x}{\sqrt{3x^2-4}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$

$u = 3x^2-4$

$du = 6x\,dx$

$\Rightarrow 2x\,dx = \frac{1}{3}du$

$= \frac{1}{3}\int u^{-1/2}\,du = \frac{2}{3}u^{1/2} + \cancel{C} = 0$

$= \frac{2}{3}\sqrt{3x^2-4} + C$

Problem 7: For the function

$$f(x,y,z) = \frac{\sqrt{2x+z}}{2y}$$

find the function values $f(1,-1,4)$ and $f(2,1,-3)$.

$$f(1,-1,4) = \frac{\sqrt{2(1)+4}}{2(-1)} = \frac{\sqrt{6}}{-2} = -\frac{\sqrt{6}}{2}$$

$$f(2,1,-3) = \frac{\sqrt{2(2)-3}}{2(1)} = \frac{\sqrt{4-3}}{2} = \frac{1}{2}$$

Problem 8: Evaluate the definite integral.

$$\int_2^4 3xe^{-x^2}\,dx$$

$u = -x^2 \Rightarrow du = -2x\,dx$

$\Rightarrow x\,dx = -\frac{1}{2}\,du$

$x = 2 \Rightarrow u = -4$

$x = 4 \Rightarrow u = -16$

$$\int_2^4 3xe^{-x^2}\,dx = -\frac{3}{2}\int_{-4}^{-16} e^u\,du$$

$$= -\frac{3}{2}e^u \Big|_{-4}^{-16}$$

$$= -\frac{3}{2}e^{-16} + \frac{3}{2}e^{-4}$$

$$= \frac{3}{2}\left(e^{-4} - e^{-16}\right) \rightsquigarrow$$ calculator from here

Problem 9: Compute the following indefinite integral using partial fraction decomposition.

$$\int \frac{3x^2-5}{(x+3)^2}dx$$

Need long division first

$$\frac{3x^2-5}{(x+3)^2} = \frac{3x^2-5}{x^2+6x+9}$$

$$\begin{array}{r} 3 \\ x^2+6x+9 \overline{)\,3x^2-5} \\ \underline{3x^2+18x+27} \\ -18x-32 \end{array}$$

$$\Rightarrow \frac{3x^2-5}{(x+3)^2} = 3 - \frac{18x+32}{x^2+6x+9}$$

$$\int \frac{3x^2-5}{(x+3)^2}dx = \int \left(3 - \frac{18x+32}{(x+3)^2}\right)dx$$

$$= 3x - \int \frac{18x+32}{(x+3)^2}dx$$

$$= 3x - \int \frac{18}{x+3}dx + \int \frac{22}{(x+3)^2}dx$$

$$= 3x - 18\ln|x+3| - 22(x+3)^{-1} + C$$

on back

Problem 10: Find all critical points of the function

$$f(x,y) = x^2 - 4xy + y^3 - 4x - 9y + 17$$

Classify the critical point as a relative minimum, relative maximum, or saddle point using the second derivative test.

This makes the problem tougher
→ It is a typo that should be y^2......

This will be corrected in the next version.

$$\frac{18x+32}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$$

$$\Rightarrow \quad 18x+32 = A(x+3) + B$$

$$= Ax + 3A + B$$

Compare coefficients

$$18 = A$$

$$32 = 3A+B = 3(18)+B$$

$$\Rightarrow \quad B = 32 - 3(18) = -22$$

$$\Rightarrow \quad \frac{18x+32}{(x+3)^2} = \frac{18}{x+3} - \frac{22}{(x+3)^2}$$