

Problem Definition

Problem 41. Verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition.

$$\begin{aligned}\text{General Solution:} & \quad y = Ce^{-2x} \\ \text{Differential Equation:} & \quad y' + 2y = 0 \\ \text{Initial Condition:} & \quad y = 3 \text{ and } x = 0\end{aligned}$$

Solution Step 1:

We start by testing the the general solution to see if the function satisfies the differential equation. We need to compute the derivative as follows.

$$y = Ce^{-2x} \quad \rightarrow \quad y' = Ce^{-2x}(-2) = -2Ce^{-2x}$$

Now, we substitute these into the left hand side of the ordinary differential equation.

$$y' + 2y = -2Ce^{-2x} + 2(Ce^{-2x}) = -2Ce^{-2x} + 2Ce^{-2x} = 0$$

Since this makes the left hand side zero which matches the right hand side of the differential equation, the function satisfies the differential equation.

Solution Step 2:

Now, we want the function to satisfy the initial condition. This means

$$y = Ce^{-2(0)} = Ce^0 = C = 3$$

So, the particular solution must be

$$y = 3e^{-2x}$$