

Problem Definition

Problem 53. Use integration to find the general solution of the differential equation

$$\frac{dy}{dx} = 3x^2$$

Solution Step 1:

All we need to do is compute the integral of the differential equation. That is,

$$\int \frac{dy}{dx} dx = \int 3x^2 dx$$

Using the fact that

$$\int \frac{dy}{dx} dx = y + C_1$$

we can integrate both sides of the integral equation above. The calculations are

$$y + C_1 = \int 3x^2 dx = 3\frac{x^3}{3} + C_2 = x^3 + C_2$$

Solution Step 2:

The last step is to do some algebra on the integrated form of the equation. Solving for y gives

$$y = x^3 + C_2 - C_1 = x^3 + C$$

where we let $C = C_2 - C_1$. So, the general solution for this differential equation is

$$y = x^3 + C$$

It is referred to as a general solution since the constant C is not known. This also means a general solution is not unique.