Problem Definition

Problem 65. **Biology** The limiting capacity of the habitat of a wildlife herd is 750. The growth rate $\frac{dN}{dt}$ of the herd is proportional to the unutilized opportunity for growth, as described by the differential equation

$$\frac{dN}{dt} = k(750 - N)$$

The general solution of this differential equation is

$$N = 750 - Ce^{-kt}$$

When $t = 0$ the population of the herd is 100. After 2 years the population has grown to 160.

(a) Write the population function N as a function of t.

(b) Use a grpahing utility to graph the population function.

(c) What is the population of the herd after 4 years?

Solution Step 1:

To write a function, we will need to determine the constant C and the rate constant k in the general solution. This is like an exponentatial growth/decay model. To start, we can consider the initial time $t = 0$ where

$$N = 750 - Ce^{-k(0)} = 750 - Ce^0 = 750 - C = 100$$

This means that $C = 750 - 100 = 650$. Also, we know that after two years when $t = 2$ the population value will be 160. This means

$$N = 750 - 650e^{-k(2)} = 160$$

Solving for k gives the following list of equations.

$$\begin{aligned} 750 - 160 &= -650e^{-2k} \\ 590 &= 650e^{-2k} \\ \frac{590}{650} &= e^{-2k} \\ ln|0.9077| &\approx -2k \\ 0.0484 &\approx k \end{aligned}$$

So, the model is

$$N = 750 - 650e^{-(0.0484)t}$$

Solution Step 2:

Use a graphing utility to graph the function obtained above.

Solution Step 3:

The population after four years can be predicted using $t = 4$ in the model.

$$N = 750 - 650e^{-(0.0484)4} \approx 214$$

There will be approximately 214 individuals in the herd with the herd continuing to increase.