Problem Definition

Problem 41. **Learning Theory:** The management of a factory has found that a worker can produce at most 30 units per day. The number of units N per day produced by a new employee will increase at a rate proportional to the difference between 30 and N. This is described by the differential equation

$$\frac{dN}{dt} = k(30 - N)$$

where t is the time in days. Solve this differential equation.

Solution Step 1:

The process of separation of variables starts by moving all dependence on t to one side of the equation and all dependence on N to the other. Multiplying the differential equation through by dt results in

$$\frac{dN}{dt}dt = k(30 - N)dt$$

or

$$dN = k(30 - N)dt$$

Dividing by $30 - N$ in trun gives

$$\frac{dN}{30 - N} = kdt$$

This shows the equation is separable.

Solution Step 2:

Next we need to integrate both sides of the separated equation. Integrating the left hand side gives

$$\int \frac{dN}{30 - N} = -ln|30 - N| + C_1$$

and

$$\int kdt = kt + C_2$$

Setting the integrals equal gives the equation

$$-ln|30 - N| + C_1 = kt + C_2$$

or

$$ln|30 - N| = -kt + C$$

Exponentiating both sides of the equation results in

$$30 - N = e^{-kt+C} = e^C \ e^{-kt}$$

Solving for N gives

$$N = 30 - e^C \ e^{-kt}$$

This is the general solution of the original differential equation. To find a particular solution we would need an initial condition and another data point or the rate constant.