**Problem Definition**

Problem 13. Solve the differential equation

$$y' + 5xy = x$$

**Solution Step 1:**

This equation is a first order linear equation and as such the first step is to compute the associated integrating factor. This is done as follows.

$$u(x) = e^{\int P(x)dx}$$

where in this problem $P(x) = 5x$. Also, keep in mind that in this application we do not need the constant of integration. Thus,

$$\int P(x)dx = \int 5x dx = \frac{5}{2}\, x^2$$

Then

$$u(x) = e^{(5/2)x^2}$$

**Solution Step 2:**

Once the integrating factor is known we can compute the solution using the formula.

$$y(x) = \frac{1}{u(x)} \int Q(x)u(x)dx$$

where in this problem $Q(x) = x$. This formula can be derived, but there is no need to do this since it works in most cases. We need to compute the integral in the formula. In this problem we need to compute

$$\begin{aligned}
\int Q(x)u(x)dx &= \int xe^{(5/2)x^2}\,dx \\
&= \frac{1}{5}\int 5xe^{(5/2)x^2}\,dx \\
&\qquad \text{substitute: with} \qquad u = (5/2)x^2 \\
&= \frac{1}{5}\int e^u du \\
&= \frac{1}{5}e^u + C \\
&= \frac{1}{5}e^{(5/2)x^2} + C
\end{aligned}$$

**Solution Step 3:**

Now that we have the pieces, the solution is

$$\begin{aligned} y(x) &= \frac{1}{e^{(5/2)x^2}}\left(\frac{1}{5}e^{(5/2)x^2} + C\right) \\ &= e^{-(5/2)x^2}\left(\frac{1}{5}e^{(5/2)x^2} + C\right) \\ &= \frac{1}{5} + Ce^{-(5/2)x^2} \end{aligned}$$

This is the general solution with $C$ to be determined by an initial condition if we need a particular solution.