Problem Definition

Problem 9. **Sales Growth** The rate of change in sales S (in thousands of units) of a new product is proportional to the difference between L and S (in thousands of units) at any time t. When $t = 0$, $S = 0$. Write and solve the differential equation for this sales model.

Solution Step 1:

The first step in this problem is to build a differential equation that models the process described in the problem. The rate of change of sales is defined by dS/dt with a proportionality relationship of the form

$$\frac{dS}{dt} \propto (L - S)$$

where the symbol $\propto$ is the proportionality symbol. To finish off the equation we can define the proportionality constant to be k and write

$$\frac{dS}{dt} = k(L - S) = -k(S - L)$$

The initial condition described in the problem is $S(0) = 0$.

Solution Step 2:

We can use separation for variables to compute the solution. The separation of variables will produce an equation of the form

$$\frac{dS}{S - L} = -kdt$$

Integrating both sides results in the form

$$ln|S - L| + C_1 = -kt + C_2$$

or we can write

$$ln|S - L| = -kt + C_2 - C_1 = -kt + C_3$$

Applying the natural exponential to both sides results in the form

$$S - L = e^{-kt+C_3} = e^{-kt}e^{C_3} = e^{-kt}C = Ce^{-kt}$$

Solving for the sales function $S(t)$ gives

$$S = L + Ce^{-kt}$$

This is the general solution of the differential equation.

Solution Step 3:

The last step is to apply the initial condition. In the problem it is stated that the value of sales is initially zero. That is,

$$S(0) = L + Ce^{-k(0)} = L + C = 0$$

This implies that $C = -L$ and the particular solution is given by the following formula

$$S(t) = L - Le^{-kt}$$

Given a value of L and another value of sales we can compute k the proportionality constant.