

Problem Definition

Problem 47. Find the limit if it exists.

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$$

Solution Step 1:

Since this is a rational function of polynomials, the first step is to evaluate each of the limits of the polynomials separately. The limit for the numerator is

$$\lim_{x \rightarrow -2} (x^3 + 8) = 0$$

meaning the numerator is approaching zero as x gets close to -2 . The limit for the denominator is the following

$$\lim_{x \rightarrow -2} (x + 2) = (-2 + 2) = 0$$

Solution Step 2:

If the limit in the denominator is zero, we need to see what the limit in the numerator is. Since this limit is also zero we cannot determine a value at this point. Instead we need to do a bit of algebra to proceed. We can factor the polynomial into the following form.

$$x^3 + 8 = (x + 2)(x^2 - 2x + 4)$$

Since a limit only considers points near $x = -2$ we can write

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = \lim_{x \rightarrow -2} \frac{(x + 2)(x^2 - 2x + 4)}{x + 2} = \lim_{x \rightarrow -2} (x^2 - 2x + 4)$$

Solution Step 3:

Since the result of the algebra is a polynomial we can write the following limit.

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = \lim_{x \rightarrow -2} (x^2 - 2x + 4) = (-2)^2 - 2(-2) + 4 = 12$$

In this case the limit exists and can be found after a bit of algebra.