

### Problem Definition

Problem 53. Find the limit if it exists.

$$\lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x) - 2x}{\Delta x}$$

### Solution Step 1:

Since this is a rational function of polynomials, the first step is to evaluate each of the limits of the polynomials separately. The limit for the numerator is

$$\lim_{\Delta x \rightarrow 0} (2(x + \Delta x) - 2x) = 0$$

meaning the numerator is approaching zero as  $x$  gets close to  $-2$ . The limit for the denominator is the following

$$\lim_{\Delta x \rightarrow 0} \Delta x = 0$$

### Solution Step 2:

If the limit in the denominator is zero, we need to see what the limit in the numerator is. Since this limit is also zero we cannot determine a value at this point. Instead we need to do a bit of algebra to proceed.

$$2(x + \Delta x) - 2x = 2x + 2\Delta x - 2x = \Delta x$$

Since a limit only considers points near  $\Delta x = 0$  we can write

$$\lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x) - 2x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{2\Delta x}{\Delta x} = \lim_{\Delta x \rightarrow 0} 2$$

where the factors of  $\Delta x$  has been cancelled.

**Solution Step 3:** Putting things together gives

$$\lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x) - 2x}{\Delta x} = \lim_{\Delta x \rightarrow 0} 2 = 2$$

In this case the limit exists, is equal to 2, and can be found after a bit of algebra.