

Problem Definition

Problem 17. Determine the intervals on which the following function is continuous.

$$f(x) = \frac{x}{x^2 - 1}$$

Solution Step 1:

Since this is a rational function of polynomials, the first step is to determine the locations where the polynomial in the denominator is zero. Factoring the denominator means we can rewrite the function in the form

$$f(x) = \frac{x}{(x + 1)(x - 1)}$$

The roots of the polynomial in the denominator are $x = -1$ and $x = 1$. At any other real number, the function is well defined and since the functions are polynomials, the limit exists and is equal to the function value.

Solution Step 2:

The next step is to work on the points where the denominator is zero. Since the function is not defined at these points, the function cannot be continuous at the two points even if the limit does exist. So, the largest set of real numbers on which the function is continuous is given by

$$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

Note that this coincides with the domain of the function.