

Problem Definition

Problem 39. Sketch the graph of the following function.

$$f(x) = \frac{x^2 - 16}{x - 4}$$

Solution Step 1:

We need to first determine the domain of the function. Since there is a zero in the denominator of the rational function at $x = 4$ the function is not defined at this point. The domain is $(-\infty, 4) \cup (4, \infty)$.

Solution Step 2:

We will need to determine if the point $x = 4$ is the location of a singularity or if the singularity is removable. To start this process, we will factor the polynomials in the rational expression as far as we can. This can be done as follows.

$$f(x) = \frac{x^2 - 16}{x - 4} = \frac{(x - 4)(x + 4)}{x - 4} = x + 4$$

provided we understand that $x = 4$ is not in the domain of the original function. This means the singularity is removable.

Solution Step 3:

The last step is to sketch the graph of the reduced form or equivalent expression $g(x) = x + 4$. with an open circle at $x = 4$ to indicate that the original function is not defined at $x = 4$.