Problem Definition

Problem 17. Use the limit definition to find the derivative of the following function.

$$f(x) = -5x + 3$$

Solution Step 1:

The limit definition is

$$f'(x) = \frac{df}{dx}(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Solution Step 2:

The limit in this particular case can be computed as follows.

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(-5(x + \Delta x) + 3) - (-5x + 3)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-5x - 5\Delta x + 3 + 5x - 3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-5\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} (-5) = -5 \end{aligned}$$

So the derivative of the function is $f'(x) = -5$.