

Problem Definition

Problem 27. Find the slope of the tangent line to the function

$$f(x) = 6 - 2x$$

at the point $(2, 2)$.

Solution Step 1:

The point $(2, 2)$ satisfies

$$f(2) = 6 - 2(2) = 6 - 4 = 2$$

To compute the slope of the tangent line we can start with the slope of the secant line between $x = 2$ and $x = 2 + \Delta x$. This slope is given by

$$m = \frac{f(2 + \Delta x) - f(2)}{(2 + \Delta x) - 2} = \frac{f(2 + \Delta x) - f(2)}{\Delta x}$$

Solution Step 2:

To find the slope of the tangent line, we will need to compute the limit as Δx tends to zero. Note that we are doing this with the definition of the derivative. So,

$$\begin{aligned} f'(2) &= \lim_{\Delta x \rightarrow 0} \frac{f(2 + \Delta x) - f(2)}{(2 + \Delta x) - 2} \\ &= \lim_{\Delta x \rightarrow 0} \frac{6 - 2(2 + \Delta x) - 2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-2\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (-2) = -2 \end{aligned}$$

So, the slope of the tangent line is a constant value of -2 .