Problem Definition

Problem 51. Determine the points at which the following function has a horizontal tangent line.

$$y = -x^4 + 3x^2 - 1$$

Solution Step 1:

The slope of the tangent line is given by the derivative evaluated at a given point. The derivative in this case is given by

$$\frac{dy}{dx} = -4x^3 + 6x$$

Solution Step 2:

Now we need to determine points where the derivative will be zero. We can proceed by factoring the polynomial that we obtained from computing the derivative. That is,

$$\frac{dy}{dx} = -4x^3 + 6x = -2x(2x^2 - 3)$$

Values of x that produce a zero above gives

$$x = 0, \quad x = \sqrt{\frac{3}{2}}, \quad x = -\sqrt{\frac{3}{2}}$$

or

$$x = 0, \quad x = \frac{\sqrt{6}}{2}, \quad x = -\frac{\sqrt{6}}{2}$$

Solution Step 3:

To finish, we can compute the point on the graph of the function that intersects the horizontal tangent line. When $x = 0$, $y = -1$ which implies that $(0, -1)$ is a point on the graph of the function where the tangent line is horizontal. The other two points are

$$(\frac{\sqrt{6}}{2}, \frac{5}{4}) \qquad \text{and} \qquad (-\frac{\sqrt{6}}{2}, \frac{5}{4})$$