Problem Definition

Problem 13. (Medicine Application) The effectiveness, E, (on a scale from 0 to 1) of a pain killing drug t hours after entering the bloodstream us given by

$$E = \frac{1}{27}(9t + 3t^2 - t^3)$$

for $0 \leq t \leq 4.5$. Find the average rates of change of E on each of the indicated intervals and compare these with the rates of change at the endpoints of the interval.

(a) $[0, 1]$

(b) $[1, 2]$

(c) $[2, 3]$

(d) $[3, 4]$

Solution Step 1:

First, we will need to compute the derivative of the function defining the effectiveness as a function of time.

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6t - 3t^2)$$

We will use this to compute the instantaneous rate of change of the effectiveness of the drug.

Solution Step 2:

On the interval $[0, 1]$ the average rate of change is given by

$$\frac{E(1) - E(0)}{1 - 0} = \frac{11}{27} - 0 = \frac{11}{27}$$

At the endpoints of the intervals the instantaneous rate of change at $t = 0$ is

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(0) - 3(0)^2) = \frac{1}{3}$$

and at $t = 1$

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(1) - 3(1)^2) = \frac{4}{9}$$

Note that the average rate of change is between the instantaneous rate of change measured at the endpoints of the interval.

Solution Step 3:

On the interval $[1, 2]$ the average rate of change is given by

$$\frac{E(2) - E(1)}{2 - 1} = \frac{22}{27} - \frac{11}{27} = \frac{11}{27}$$

At the endpoints of the intervals the instantaneous rate of change at $t = 1$ is

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(1) - 3(1)^2) = \frac{4}{9}$$

and at $t = 2$

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(2) - 3(2)^2) = \frac{1}{3}$$

Again note that the average rate of change is between the instantaneous rate of change measured at the endpoints of the interval.

Solution Step 4:

On the interval $[2, 3]$ the average rate of change is given by

$$\frac{E(3) - E(2)}{3 - 2} = \frac{27}{27} - \frac{22}{27} = \frac{5}{9}$$

At the endpoints of the intervals the instantaneous rate of change at $t = 2$ is

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(2) - 3(2)^2) = \frac{1}{3}$$

and at $t = 3$

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(3) - 3(3)^2) = 0$$

In this case the average rate of change is larger than the instantaneous rate of change measured at the endpoints of the interval.

Solution Step 5:

On the interval $[3,4]$ the average rate of change is given by

$$\frac{E(4)-E(3)}{4-3} = \frac{20}{27} - \frac{27}{27} = \frac{5}{9}$$

At the endpoints of the intervals the instantaneous rate of change at $t = 3$ is

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(3) - 3(3)^2) = 0$$

and at $t = 4$

$$\frac{dE}{dt} = \frac{1}{27}(9 + 6(4) - 3(4)^2) = -\frac{5}{9}$$

In this case the average rate of change is between than the instantaneous rate of change measured at the endpoints of the interval.