Problem Definition

Problem 9. Graph the following function on the given interval and compare the average rate of change to the instantaneous rate of change of the function at the endpoints of the interval.

$$f(x) = x^4 - x^2 + 2 \qquad [1, 3]$$

Solution Step 1:

First use your graphing calculator or computer graphics application to graph the function.

Solution Step 2:

The average rate of change is given by computing the difference in function values at the ends of the interval and dividing by the length of the interval. That is,

$$\begin{aligned} \text{Average Rate of Change} &= \frac{f(3) - f(1)}{3 - 1} = \frac{(81 - 9 + 2) - (1 - 1 + 2)}{2} \\ &= \frac{72}{2} = 36 \end{aligned}$$

Solution Step 3:

To compute the instantaneous rate of change of the function requires the computation of the derivative of the function. The derivative is given by

$$f'(x) = 4x^3 - 2x$$

The instantaneous rate of change at the two points is given by

$$f'(1) = 4(1)^3 - 2(1) = 2$$

and

$$f'(3) = 4(3)^3 - 2(3) = 102$$

The important point to note is that the average rate of change is between the instantaneous rate of change at the two endpoints of the interval.