

Problem Definition

Problem 31. Find the derivative of the following function.

$$f(x) = \frac{3 - 2x - x^2}{x^2 - 1}$$

Solution Step 1:

In this case, we have a quotient of two polynomials. So, we can use the quotient rule to compute the derivative. The general formula can be written as

$$\frac{d}{dx} \frac{g(x)}{h(x)} = \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$$

For our problem, we compute the derivative as follows.

$$\begin{aligned} f'(x) &= \frac{d}{dx} \frac{3 - 2x - x^2}{x^2 - 1} \\ &= \frac{(-2 - 2x)(x^2 - 1) - (2 - 2x - x^2)(2x - 0)}{(x^2 - 1)^2} \\ &= \frac{(-2 - 2x)(x^2 - 1) - (2 - 2x - x^2)(2x - 0)}{(x^2 - 1)^2} \\ &= \frac{2x^2 - 4x + 2}{(x^2 - 1)^2} \\ &= \frac{2(x^2 - 2x + 1)}{(x + 1)^2(x - 1)^2} \\ &= \frac{2(x - 1)^2}{(x + 1)^2(x - 1)^2} \\ &= \frac{2}{(x + 1)^2} \end{aligned}$$

This comes from

$$g(x) = 3 - 2x - x^2$$

and

$$h(x) = x^2 - 1$$

for the general formula above. In this solution we have simplified things a bit to match the solution that appears in the book.