

Problem Definition

Problem 41. Find the equation of the tangent line to the function at the given point

$$f(x) = \frac{x-2}{x+1} \quad (1, -1/2)$$

Then use a graphing calculator or graphing utility to graph the function and the tangent line in a neighborhood of the point given.

Solution Step 1:

The first step is to compute the derivative of the function.

$$f'(x) = \frac{(1)(x+1) - (x-2)(1)}{(x+1)^2} = \frac{3}{(x+1)^2}$$

The derivative at the point given is

$$f'(1) = \frac{3}{(1+1)^2} = \frac{3}{4}$$

This gives the slope of the tangent line.

Solution Step 2:

The form of the linear function is of the form

$$y - y_0 = m(x - x_0)$$

with $m = 3/4$ and the point is $(x_0, y_0) = (1, -1/2)$. The equation of the tangent line is

$$y - (-\frac{1}{2}) = \frac{3}{4}(x - 1)$$

Simplifying gives the linear equation

$$y = \frac{3}{4}x - \frac{5}{4}$$

Solution Step 3:

The last step in the process is to graph the two functions on the same plot to see how the equation is really tangent to the graph of the function.