

Problem Definition

Problem 57. **Population Growth** A population of bacteria is introduced into a culture. The number of bacteria P can be modeled by

$$P = 500 \left(t + \frac{4t}{50 + t^2} \right)$$

where t is the time measured in hours. Find the rate of change of the population when $t = 2$ hours.

Solution Step 1:

To get the instantaneous rate of change, we need to compute the derivative. This is determined as follows

$$\begin{aligned} \frac{dP}{dt} &= 500 \frac{d}{dt} \left(t + \frac{4t}{50 + t^2} \right) \\ &= 500 \left(1 + \frac{4(50 + t^2) - (4t)(2t)}{(50 + t^2)^2} \right) \end{aligned}$$

We do not need to simplify this expression to compute the value at $t = 2$. The rate of change is

$$\frac{dP}{dt} = 500 \left(1 + \frac{4(50 + (2)^2) - (4(2))(2(2))}{(50 + (2)^2)^2} \right) \approx 31.55$$

in units of bacteria per hour.