Problem Definition

Problem 55. Compute the derivative of the function.

$$y = x\sqrt{2x+3}$$

Solution Step 1:

The function can be rewritten to allow an easier application of our differentiation rules.

$$y = x\sqrt{2x+3} = x(2x+3)^{\frac{1}{2}}$$

Solution Step 2:

The derivative will be computed using the product rules followed by an application of the generalized power rule on the square root factor in the function on the factors to start the process. The intermediate result of our work is the following.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(x(2x+3)^{\frac{1}{2}}\right) \\
&= (1)(2x+3)^{\frac{1}{2}} + x\frac{d}{dx}(2x+3)^{\frac{1}{2}} \\
&= (2x+3)^{\frac{1}{2}} + x\frac{1}{2}(2x+3)^{-\frac{1}{2}}\frac{d}{dx}(2x+3) \\
&= (2x+3)^{\frac{1}{2}} + x\frac{1}{2}(2x+3)^{-\frac{1}{2}}(2) \\
&= (2x+3)^{\frac{1}{2}} + x(2x+3)^{-\frac{1}{2}} \\
&= \frac{(2x+3)+x}{(2x+3)^{\frac{1}{2}}} \\
&= \frac{3(2x+1)}{(2x+3)^{\frac{1}{2}}}
\end{aligned}$$

which is the solution given in the book.