Problem Definition

Problem 73. **Biology** The number of bacteria in a culture after t days is modeled by

$$N(t) = 400\left[1 - \frac{3}{(t^2+2)^2}\right]$$

Compute the table and discuss what can be concluded from the data you collect.

t	0	1	2	3	4
$N'(t)$					

Solution Step 1:

To start the work we will need the derivative of the function with respect to time.

$$\begin{aligned} N'(t) &= 400\frac{d}{dt}\left[1 - 3(t^2+2)^{-2}\right] \\ &= 400\left[0 - (-6)(t^2+2)^{-3}(2t)\right] \\ &= 400\left[\frac{12t}{(t^2+2)^3}\right] \\ &= \frac{4800t}{(t^2+2)^3} \end{aligned}$$

Solution Step 2:

Evaluating the derivative obtained in the previous step, the filled in table looks like the following to two digits in the approximation.

t	0	1	2	3	4
$N'(t)$	0	177.78	44.44	10.29	3.29

The data indicates that the rate of change of the number of bacteria is decreasing to zero. The positive derivative indicates that the number of bacteria is increasing and eventually levels off at some value.

Using the idea of horizontal slopes, we can obtain what the value would be as t gets very large. This material is in Section 3.6 of the text on asymptotes.