## Problem Definition

Problem 11. Find $dy/dx$.

$$\frac{x+y}{2x-y} = 1$$

## Solution Step 1:

We can compute the derivative in a couple of ways. The first way is to simplify the expression before performing the differentiation or apply the quotient rule to the rational function on the left hand side. To show the process let's compute derivative of the expression as it appears.

$$\frac{d}{dx}\frac{x+y}{2x-y} = \frac{d}{dx}1$$

Applying the quotient rule to the rational function on the left hand side of the equation.

$$\frac{(1+\frac{dy}{dx})(2x-y)-(x+y)(2-\frac{dy}{dx})}{(2x-y)^2} = 0$$

## Solution Step 2:

Assuming that the denominator is not zero, we can write

$$(1+\frac{dy}{dx})(2x-y)-(x+y)(2-\frac{dy}{dx}) = 0$$

or using the $y'$ notation

$$(1+y')(2x-y)-(x+y)(2-y') = 0$$

Next, if we expand the expressions

$$(2x-y+2xy'-yy')-(2x-xy'+2y-yy') = 0$$

or simplfying gives the equation

$$-3y+3xy' = 0$$

This turns into the form

$$y' = \frac{dy}{dx} = \frac{3y}{3x} = \frac{y}{x}$$

**Solution Step 3:**

So, how does the book end up with $dy/dx = 1/2$. If we return to the original equation, we can solve for $y$ in terms of $x$. Multiplying the equation by $2x - y$ will result in the equation

$$x + y = 2x - y$$

subtracting $x$ from both sides and adding $y$ to both sides results in the equation

$$2y = x$$

Now, let's substitute this into the derivative we obtained.

$$\frac{dy}{dx} = \frac{y}{2y} = \frac{1}{2}$$

dividing out the common factor of $y$. So, we can retrieve the constant value with a bit of algebra.