**Problem Definition**

Problem 37. Find equations of tangent lines to the graph of the solution set of the equation given below at the given points.

$$y^2 = 5x^2 \qquad (1, \sqrt{5}) \text{ and } (1, -\sqrt{5})$$

**Solution Step 1:**

We will need the slope of the tangent lines at the given points. So, we need to compute the derivative via implicit differentiation. Differentiation of both sides of the equation gives

$$\frac{d}{dx}y^2 = \frac{d}{dx}(5x^3)$$

Computing the derivatives gives

$$2y\frac{dy}{dx} = 15x$$

or solving gives

$$\frac{dy}{dx} = \frac{15x}{2y}$$

We will be able to use this expression to compute the slope of the tangent lines.

**Solution Step 2:**

The form of the tangent line comes from the point-slope form shown below.

$$y - y_0 = m(x - x_0)$$

For the first point $(x_0, y_0) = (1, \sqrt{5})$, the slope is given by

$$m = f'(1) = \frac{15(1)}{2\sqrt{5}} = \frac{3}{2}\sqrt{5}$$

Substitution of the values into the linear equation gives

$$y - \sqrt{5} = \frac{3}{2}\sqrt{5}(x - 1)$$

To get closer to the answer in the text multiply by $\sqrt{5}$ to rewrite the equation

$$\sqrt{5}y - 5 = \frac{15}{2}(x - 1)$$

and then multiply both sides of the equation by 2 to obtain

$$2\sqrt{5}y - 10 = 15x - 15$$

Moving everything over to the left side of the equation by subtracting $2\sqrt{5}$ from both sides of the equation to obtain

$$0 = 15x - 2\sqrt{5}y - 5$$

The other point will result in an equation of the form

$$0 = 15x + 2\sqrt{5}y - 5$$

due to the change in the $y$ coordinate.