**Problem Definition**

Problem 43. **Demand:** Compute the rate of change of $x$ (the demand) with respect to $p$ (the price).

$$p = \sqrt{\frac{200 - x}{2x}}$$

for $0 < x \leq 200$.

**Solution Step 1:**

In this problem we want to find the instantaneous rate of change in the demand, $x$ as a function of the price $p$ being charged for a product. This means we need to compute $dx/dp$. It is best to do a little algebra on this problem before using implicit differentiation. Squaring both sides of the equation gives

$$p^2 = \frac{200 - x}{2x}$$

Multiplying both sides by $2x$ gives

$$2xp^2 = 200 - x$$

Finally, if we add $x$ to both sides of the equation, we end up with

$$2xp^2 + x = 200$$

**Solution Step 2:**

Using implicit differentiation on the equation gives

$$\frac{d}{dp}(2xp^2 + x) = \frac{d}{dp}(200)$$

which produces

$$2\frac{dx}{dp}p^2 + 2x(2p) + \frac{dx}{dp} = 0$$

or

$$2\frac{dx}{dp}p^2 + 4xp + \frac{dx}{dp} = 0$$

**Solution Step 3:**

Now we need to solve for $dx/dp$ using a little bit of algebra. First, subtracting $4xp$ from both sides gives

$$(2p^2 + 1)\frac{dx}{dp} = -4xp$$

and then solving for $dx/dp$ gives

$$\frac{dx}{dp} = \frac{-4xp}{(2p^2 + 1)}$$

This is an implicit expression for the rate of change of the demand with respest to the price of the items produced.