Problem Definition

Problem 11. **Cost, Revenue, and Profit** A company that produces sports supplements calculates that it costs and revenue can be modeled by the equations

$$C = 125,000 + 0.75x \qquad R = 250x - \frac{1}{10}x^2$$

where x is the numebr of units of sport supplements produced in a week. If production in one particular week is 1000 units and is increasing at a rate of 150 units per week, find:

(a) the rate at which the cost is changing,

(b) the rate at which the revenue is changing, and

(x) the rate at which the profit is changing.

Solution Step 1:

For part (a) we need to compute the derivative of both sides of the cost equation with respect to time, t. This gives

$$\frac{dC}{dt} = 0.75\frac{dx}{dt}$$

Using the rate of change in production gives

$$\frac{dC}{dt} = 0.75(150) = 112.25$$

dollars per week.

Solution Step 2:

For part (b) we need to compute the derivative of both sides of the cost equation with respect to time, t. This gives

$$\frac{dR}{dt} = 250\frac{dx}{dt} - \frac{1}{5}x\frac{dx}{dt}$$

Using the rate of change in production gives

$$\frac{dR}{dt} = 250(150) - \frac{1}{5}(1000)(150) = 7500$$

dollars per week.

Solution Step 3:

For part (c) we know the profit is the revenue minus the cost. So, we can write

$$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt}$$

So, we can use the results from parts (a) and (b)

$$\frac{dP}{dt} = 7500 - 112.5 = 7387.5$$

dollars per week.