

Problem Definition

Problem 21. **Athletics** A square baseball diamond has sides that are 90 feet long. A player 26 feet from third base is running at a speed of 30 feet per second. At what rate is the distance from the player to home plate changing?

Solution Step 1:

The line between the player and home plate forms the hypotenuse of a right triangle. One leg of the triangle is the line from third base to home plate and the other leg is the line from the player to third base. The distance between the player and home plate is given by the Pythagorean formula relating the lengths of the sides of a right triangle. If x is the distance between the player and third base then we can write

$$90^2 + x^2 = h^2$$

h is the distance between the player and home plate.

Solution Step 2:

Applying implicit differentiation we obtain

$$\frac{d}{dt}(90^2 + x^2) = \frac{d}{dt}h^2$$

or

$$0 + 2x \frac{dx}{dt} = 2h \frac{dh}{dt}$$

To complete the problem, we need to compute the distance h when $x = 26$. The original formula implies

$$h = \sqrt{90^2 + 26^2} = \sqrt{8776} = 93.68$$

This means

$$2(26)(30) = 2(93.68) \frac{dh}{dt}$$

Solving for dh/dt gives

$$\frac{dh}{dt} = -8.33$$

feet per second. The minus sign is needed since the distance from the player to home base is getting smaller.