Problem Definition

Problem 15. Compute the critical numbers and the open intervals on which the function is increasing or decreasing. Sketch the graph of the function.

$$y = x^3 - 6x^2$$

Solution Step 1:

The first step is to compute the derivative of the of the function.

$$\frac{dy}{dx} = 3x^2 - 12x = 3x(x - 4)$$

Solution Step 2:

With the derivative computed, the next step is to identify the critical points. There are two types of critical points; points of discontinuity and points where the derivative is zero. Since the derivative is a polynomial, there will be no points of continuities. The only critical points are where the derivative is zero. So, we find x such that

$$3x(x - 4) = 0$$

The two zeros are $x = 0$ and $x = 4$.

Solution Step 3:

Between the critical points the function will be either increasing or decreasing. So, we test points between the critical points. The intervals can be analyzed one by one.

(a) The first interval is $(-\infty, 0)$. For $x = -1$, the derivative is

$$y'(-1) = 3(-1)(-1 - 4) = 15 > 0$$

Since $y'(x) > 0$ for this test point, the function is increasing for the interval $(-\infty, 0)$.

(b) Next we need to test $(0, 4)$. For $x = 1$, the derivative is

$$y'(1) = 3(1)(1 - 4) = -9 < 0$$

Since $y'(x) < 0$ for this test point, the function is decreasing for the interval $(0, 4)$.

(c) Finally, we need to test $(4, \infty)$. For $x = 1$, the derivative is

$$y'(5) = 3(5)(5 - 4) = 15 > 0$$

Since $y'(x) > 0$ for this test point, the function is increasing for the interval $(4, \infty)$.

Solution Step 4:

Summarizing, we can write:

(a) The function $y(x)$ is increasing on $(-\infty, 0)$ and $(0, \infty)$.

(b) The function $y(x)$ is decreasing on $(0, 4)$.