Problem Definition

Problem 31. Find the critical numbers and the open intervals on which the function is increasing or decreasing. Check for discontinuities and sketch the graph.

$$y = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$$

Solution Step 1:

The first step in this case is to look at the point $x = 0$. The one sided limits are

$$\lim_{x \to 0^-} y(x) = \lim_{x \to 0^-} (4 - x^2) = 4$$

and

$$\lim_{x \to 0^+} y(x) = \lim_{x \to 0^-} 2x = 0$$

Since the limits are not equal, the function is not continuous at $x = 0$.

Solution Step 2:

The next step is to compute the derivative of the of the function away from the change in the function at $x = 0$.

$$y = \begin{cases} -2x, & x \le 0 \\ -2, & x > 0 \end{cases}$$

Solution Step 3:

In this case, we still need to find where the derivative is zero away from the points of discontinuity. For this function neither $2x$ or the constant 2 is zero *away* from $x = 0$. So, the only critical point is $x = 0$.

Solution Step 4:

Between the critical points the function will be either increasing or decreasing. So, we test points between the critical points. The intervals can be analyzed one by one.

(a) The first interval is $(-\infty, 0)$. For $x = -1$, the derivative is

$$y'(-1) = -2(-1) = 2 > 0$$

Since $y'(x) > 0$ for this test point, the function is increasing for the interval $(-\infty, 0)$.

(b) Next we need to test $(0, \infty)$. For $x = 1$, the derivative is

$$y'(1) = -2 < 0$$

Since $y'(x) < 0$ for this test point, the function is decreasing for the interval $(0, \infty)$.

Solution Step 5:

Summarizing, we can write:

(a) The function $y(x)$ is increasing on $(-\infty, 0)$ and

(b) The function $y(x)$ is decreasing on $(0, \infty)$.