Problem Definition

Problem 39. **Law Degrees:** The number y of law degrees conferred in the United States from 1970 to 2000 can be modeled by

$$y = 2.743t^3 - 171.55t^2 + 3462.3t + 15,265$$

where t is time in years with $t = 0$ corresponding to 1970.

(a) Use a graphing utility to graph the model. Then graphically estimate the years during in which the model is increasing and the years in which the model is decreasing.

(b) Use the first derivative test to verify the results from part (a).

Solution Step 1:

For the first part, use a graphing calculator to graph the function to see how the function behaves over the appropriate interval.

Solution Step 2:

The second part requires that we do the analysis of the function from our knowledge of calculus. The first step is to compute the derivative of the function. The function is

$$y' = 8.229t^2 - 343.10t + 3462.3$$

Solution Step 3:

Since the function is a polynomial and thus is continuous, we only need to look for roots of the polynomial.

$$8.229t^2 - 343.10t + 3462.3 = 0$$

The roots can be obtained using the quadratic formula. This gives

$$t = \frac{343.10 \pm \sqrt{(343.10)^2 - 4(8.229)(3462.3)}}{(2)(8.229)}$$

The two roots are approximately, $t \approx 17.125$ and $t \approx 24.569$. These are approximations for the critical points of the function.

Solution Step 4:

There are three intervals to analyze in this case. These are listed below. be analyzed one by one.

(a) The first interval is $(-\infty, 17.125)$. For $x = 0$, the derivative is

$$y'(0) = 8.229(0)^2 - 343.10(0) + 3462.3 = 3462.3 > 0$$

Since $y'(x) > 0$ for this test point, the function is increasing for the interval $(-\infty, 17.125)$.

(b) Next we need to test $(17.125, 24.569)$. For $x = 20$, the derivative is

$$y'(0) = 8.229(20)^2 - 343.10(20) + 3462.3 \approx -108.1 < 0$$

Since $y'(x) < 0$ for this test point, the function is decreasing for the interval $(17.125, 24.569)$.

(b) Next we need to test $(24.569, \infty)$. For $x = 30$, the derivative is

$$y'(0) = 8.229(30)^2 - 343.10(30) + 3462.3 \approx 575.4 > 0$$

Since $y'(x) > 0$ for this test point, the function is increasing for the interval $(24.569, \infty)$.

Solution Step 4:

To Summarize, we can write:

(a) The function $y(x)$ is increasing on $(-\infty, 17.125)$ and $(24.569, \infty)$.

(b) The function $y(x)$ is decreasing on $(17.125, 24.569)$.

This should agree (at least approximately) with the graph from the first part.