Problem Definition

Problem 11. Find the relative extrema of the following function.

$$f(x) = x^4 - 2x^3$$

Solution Step 1:

We start by computing the derivative of the function.

$$f'(x) = 4x^3 - 6x^2 = 2x^2(2x - 3)$$

Solution Step 2:

Using the derivative the next step is to identify the critical points. Since the function is a polynomial, the function and the derivative (another polynomial) are continuous for all real numbers. The only critical points are the zeros of the derivative and are found by setting the derivative to zero.

$$2x^2(2x - 3) = 0$$

The roots and thus the critical points are $x = 0$ and $x = 3/2$.

Solution Step 3:

Using the first derivative test requires an interval sign analysis between the critical points.

$$2x^2(2x - 3) = 0$$

The roots and thus the critical points are $x = 0$ and $x = 3/2$.

Solution Step 4:

The interval analysis is contained in the following list.

(a) For the interval $(-\infty, 0)$ the derivative at $x = -1$ is

$$y'(-1) = 2(-1)^2(2(-1) - 3) = -10 < 0$$

Since $y'(x) < 0$ the function is decreasing on the interval $(-\infty, 0)$.

(b) Next, on the interval $(0, 3/2)$ the derivative at $x = 1$ is

$$y'(1) = 2(1)^2(2(1) - 3) = -2 < 0$$

Since $y'(x) < 0$ the function is decreasing on the interval $(0, 3/2)$.

(c) Finally, on the interval $(3/2, \infty)$ the derivative at $x = 2$ is

$$y'(2) = 2(2)^2(2(2) - 3) = 8 > 0$$

Since $y'(x) > 0$ the function is increasing on the interval $(3/2, \infty)$.

Solution Step 5:

Since the goal is to determine the type of critical points we have found, the following needs to be done.

(a) The critical point $x = 0$ is neither a relative minimum or relative maximum since the derivative has the same sign (negative) on both sides of the critical point.

(b) The critical point $x = 3/2$ the derivative changes sign from negative to positive, so this critical point is the location of a relative minimum. The relative minimum value is given byo

$$f(3/2) = (3/2)^4 - 2(3/2)^3 = \frac{81}{16} - \frac{54}{8} = \frac{81}{16} - \frac{108}{8} = -\frac{27}{16}$$