Problem Definition

Problem 43. **Cost:** A retailer has determined the cost C for ordering and sorting x units of a product to be modeled by

$$C = 3x + \frac{20,000}{x}, \qquad 0 < x \leq 200$$

The delivery truck can bring at most 200 units per order. Find the order size that will minimize the cost. Use a graphing utility to verify your result.

Solution Step 1:

We start by computing the derivative of the function.

$$C' = 3 - \frac{20,000}{x^2}$$

Solution Step 2:

The critical points are $x = 0$ since the function and derivative are both discontinuous there and where

$$x^2 = \frac{20,000}{3}$$

or $x \approx 81.65$.

Solution Step 3:

Using the first derivative test requires an interval sign analysis between the critical points. As x gets closer to 0 the value of C gets arbitraily large due to the division by $x \approx 0$. There are two other points to consider. These are the critical point $x \approx 81.65$ and the right end point of the interval.

Solution Step 4:

There are a couple of ways to determine the type of points. The easiest in this case is to use the firt derivative test.

(a) The derivative of C to the left of $x = 81.65$ can be tested by testing the point $x = 10$.

$$C'(10) = 3 - \frac{20,000}{(10)^2} = 3 - 200 < 0$$

Since $C'(x) < 0$ the function is decreasing on the interval $(0, 81.65)$.

(b) On the other interval $(81.65, \infty)$ the derivative at $x = 100$ is

$$C'(100) = 3 - \frac{20,000}{(100)^2} = 3 - 2 > 0$$

Since $C'(x) > 0$ the function is increasing on the interval $(81.65, \infty)$.

This means the point $x \approx 81.65$ is a local (and absolute) minimum for the cost function. Whether 81 or 82 is chosen will be up to the person in charge.