Problem Definition

Problem 15. Find the relative extrema for the given function and use the second derivative test to verify the results when applicable.

$$f(x) = \sqrt{x^2+1}$$

Solution Step 1:

The first step is to compute the derivative of the of the function.

$$f'(x) = \frac{1}{2}(x^2+1)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+1}}$$

Solution Step 2:

The next step is to identify critical points. The function is continuous and since x^2+1 is never zero, the derivative also has no points of dicontinuity. The only critical point occurs when $x = 0$ since

$$f'(0) = \frac{0}{\sqrt{(0)^2+1}} = 0$$

Solution Step 3:

So, let's try the second derivative test on the function. The second derivative is

$$\begin{aligned} f''(x) &= \frac{d}{dx}\frac{x}{\sqrt{x^2+1}} \\ &= \frac{\sqrt{x^2+1} - x^2(x^2+1)^{-1/2}}{x^2+1} \\ &= \frac{x^2+1-x^2}{(x^2+1)^{3/2}} = \frac{1}{(x^2+1)^{3/2}} \end{aligned}$$

The quotient rule was used to compute the seccond derivative.

Solution Step 4:

The second derivative at $x = 0$ is

$$f''(0) = \frac{1}{((0)^2+1)^{3/2}} = 1 > 0$$

Since the second derivative is positive, the point must be a relative minimum.