Problem Definition

Problem 51. **Point of Diminishing Return:** Identify the point of diminishing return for the input-output function. For each function, R, is the revenue and x is the amount spend on advertising. Use a graphing utility to verify your results.

$$R = \frac{1}{50,000}(600x^2 - x^3), 0 \leq x \leq 500$$

Solution Step 1:

The first step is to compute the first and second derivatives of the of the revenue function, R.

$$R' = \frac{1}{50,000}(1200x - 3x^2) = \frac{3}{50,000}(400x - x^2)$$

and

$$R'' = \frac{3}{50,000}(400 - 2x) = \frac{3}{25,000}(200 - x)$$

Solution Step 2:

Next compute the point where the concavity of the revenue function changes. This occurs when $R''(x) = 0$. This happens when

$$200 - x = 0$$

or when $x = 200$. Note that the concavity indeed changes since

$$R'' = \frac{3}{25,000}(200 - 100) > 0, \qquad \text{and} \qquad \frac{3}{25,000}(200 - 300) < 0$$

Solution Step 3:

To summarize, the point $(200, R(200)) = (200, 320)$ is an inflection point for the revenue and is the point of diminshing returns for the company.