Problem Definition

Problem 35. **Minimum Area:** The combined perimeter of a circle and a square is 16. Find the Find the dimensions of the sqare and circle that minimizes the total area.

Solution Step 1:

The first step is define the variables used to measure the dimensions. Let's use

Variable for the lenght of one side of the square	x
Variable for the radius of the circle	r

The formula for the perimeter is given by

$$P = 4x + 2\pi r = 16$$

and the formula for the total area of the two objects is

$$A = x^2 + \pi r^2$$

Solution Step 2:

Using the perimeter formula we can solve for either x or r. It is probably a bit easier to solve for x to find

$$x = \frac{1}{4}(16 - 2\pi r) = 4 - \frac{1}{2}\pi r$$

Substituting this solution into the area formula gives the following.

$$A = (4 - \frac{1}{2}\pi r)^2 + \pi r^2 = 16 - 4\pi r + (\frac{1}{4}\pi^2 + \pi)r^2$$

Solution Step 3:

The next step is to find critical points for the area function by setting the derivative of the area with respect to r. The derivative is

$$A' = -4\pi + 2(\frac{1}{4}\pi^2 + \pi)r = -4\pi + (\frac{1}{2}\pi^2 + 2\pi)r = 0$$

There is one solution which is

$$r = \frac{8}{\pi + 4}$$

This means that the length of a side of the square must be

$$x = \frac{16}{\pi + 4}$$

Solution Step 4:

In this case, we can use the second derivative test to make sure that these dimensions minimize the total area. The second deriative of the are function is

$$A'' = 2(\frac{1}{4}\pi^2 + \pi)$$

which is a positive constant. Since $A'' > 0$ the critical point is a local minimum for the area.