Problem Definition

Problem 40. **Farming:** A strawberry farmer will receive \$4 per bushel of strawberries during the first week of harvesting. Each week after that, the value will drop \$0.10 per bushel. The farmer estimates that there are approximately 120 bushels of strawberries in the field and is increasing at a rate of four bushels per week. When should the farmer harvest the strawberries to maximize there value? How many bushels of strawberries will yield the maximum value? What is the maximum value of the strawberries?

Solution Step 1:

One way to define variables for this problem is the following.

The number of bushels of strawberries	x
The price for a bushel of strawberries	p
The time to harvest the strawberries	t
The revenue generate by the strawberries	R

The formula for the price is given by

$$p = 4 - (0.10)t$$

where the second term keeps track of the drop in price. The formula for the number of bushels is the following.

$$x = 120 + 4t$$

with 120 the initial total harvest and the $4t$ term modeling the growing strawberries. The revenue formula is

$$R = xp + (120 + 4t)(4 - (0.10)t) = 480 + 4t - 0.40t^2$$

The goal is to maximize the revenue generated from the crop.

Solution Step 2:

The next step is to compute the derivative of the revenue function

$$R' == 4 - 0.80t$$

and set this equal to zero to find the criical points. The only critical point is $t = 5$.

Solution Step 3:

The second derivative of the revenue is

$$R'' == -0.80 < 0$$

This is true for any t which means the critical point identifies an absolute maximum for the revenue function.

Solution Step 4:

Hmmm... The book says that the harvest should occur at $t = 6$. The reason is that the price stays the same for the entire week. So, the farmer might as well wait until the end of the week to harvest. This will mean a larger harvest and more revenue. The reason for this discrepency is that we are using a continuous model for a discrete change in the price. This corresponds to $t = 6$. The values we will see are the following

Time for the harvest	$t = 6$
Size of harvest	$x = 140$ bushels
Price for harvest	$p = \$3.50$ per bushels
Total Revenue for harvest	$R = \$490$

Solution Step 5:

The maximum volume of 11664 cubic inches is obtained for the dimensions $x = 18$ and $w = 36$.