

Problem Definition

Problem 27. Find the limit if it exists.

$$\lim_{x \rightarrow \infty} \frac{3x}{4x^2 - 1}$$

Solution Step 1:

In this problem, we are trying to determine the limit as x tends to ∞ . To compute the limit we can test the degree of the polynomial in the numerator against the degree in the denominator. In this case the polynomial in the numerator is a linear polynomial and thus of degree one. The polynomial in the denominator is a quadratic and is of degree two. In this case, as $x \rightarrow \infty$ the quadratic term will get larger faster.

Solution Step 2:

To see the relationship we can perform the following algebra.

$$\lim_{x \rightarrow \infty} \frac{x}{x} \frac{3}{4x - 1/x} = \lim_{x \rightarrow \infty} \frac{3}{4x - 1/x}$$

The last term in the denominator will tend to zero and then $4x$ will dominate the constant value 3. So, the limit will be zero. That is,

$$\lim_{x \rightarrow \infty} \frac{3x}{4x^2 - 1} = 0$$

Note that this also implies that the function has a horizontal asymptote of zero.