Problem Definition

Problem 45. **Demand:** The demand function for a product is modeled by

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

Find the price of the product if the quantity demanded is (a) $x = 100$ units and (b) $x = 500$ units. What is the limit of the price as x increases without bound.

Solution Step 1:

For the two demand values, we need to evaluate the expression above. For the first value (a) $x = 100$ we can write

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002(100)}}\right) = 5000\left(1 - \frac{4}{4 + e^{-0.2}}\right) \approx 849.53$$

and for the second value

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002(500)}}\right) = 5000\left(1 - \frac{4}{4 + e^{-1.0}}\right) \approx 421.12$$

The value seems to be decreasing.

Solution Step 2:

To answer the question about the value as the demand increases, we can take the limit as $x \to \infty$. That is, see what the horizontal limit would be. This is computed using information from Section 3.6.

$$\lim_{x\to\infty} 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right) = 5000 \lim_{x\to\infty}\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

Since the function is continuous, as $x \to \infty$

$$\lim_{x\to\infty} e^{-0.002x} = 0$$

This means that

$$\begin{aligned} 5000 \lim_{x\to\infty}\left(1 - \frac{4}{4 + e^{-0.002x}}\right) &= 5000 \lim_{x\to\infty}\left(1 - \frac{4}{4 + 0}\right) \\ &= 5000 \lim_{x\to\infty}(1 - 1) \\ &= 5000 \lim_{x\to\infty} 0 = 0 \end{aligned}$$

So, the price will go to zero as the demand gets arbitrarily large.