

**Problem Definition**

Problem 21. Determine an equation of the tangent line to the function at the given point.

$$y = (e^{2x} + 1)^3 \quad (0, 8)$$

**Solution Step 1:**

The first step is to apply the generalized power/chain rule to the function to compute the derivative.

$$\begin{aligned} y' &= \frac{d}{dx}(e^{2x} + 1)^3 \\ &= (3)(e^{2x} + 1)^2 \frac{d}{dx}(e^{2x} + 1) \\ &= 3(e^{2x} + 1)^2 e^{2x}(2) \\ &= 6e^{2x}(e^{2x} + 1)^2 \end{aligned}$$

**Solution Step 2:**

Next, we need the slope of the tangent line. This is the derivative evaluated at  $x = 0$ . The value is

$$y'(0) = 6e^{2(0)}(e^{2(0)} + 1)^2 = 6(1)((1) + 1)^2 = 6(2)^2 = 24$$

So, the slope is 24.

**Solution Step 3:**

The point-slope form of the tangent line equation is

$$y - y_0 = m(x - x_0)$$

where  $(x_0, y_0) = (0, 8)$  and  $m = f'(0) = 24$ . The tangent line equation is

$$y - 8 = (24)(x - 0)$$

or

$$y = 24x - 8 = 8(3x - 1)$$

This linear function can be used to approximate the original function near the point  $(0, 8)$ .