Problem Definition

Problem 37. **Depreciation** The value V (in dollars) of an item is a function of time t (in years) given by

$$V = 15,000e^{-0.6286t}$$

(a) Sketch the function over the interval $[0, 10]$. Use a graphing utility to verify your graph.

(b) Find the rate of change of V at $t = 1$.

(c) Find the rate of change of V at $t = 5$.

(d) Use the values $(0, V(0))$ and $(10, V(10))$ to find the linear depreciation model for the item.

(e) Compare the exponential function and the model from part (d). What are the advantages of each?

Solution Step 1:

Use a graphing calculator or other graphing utility to sketch the function. It should approach zero as t gets larger.

Solution Step 2:

For part (b) the first step is to compute the derivative of the function. That is,

$$\frac{dV}{dt} = 15000e^{-0.6286t}(-0.6286) = -9429e^{-0.6286t}$$

To complete this part, we need to evaluate the derivative at $t = 1$. So the rate of change at $t = 1$ is

$$\text{rate of change } = -9429e^{-0.6286(1)} \approx -5028.84$$

Solution Step 3:

For part (c) we just need to evaluate the derivative at $t = 5$ since we computed the derivative in part (b).

$$\text{rate of change } = -9429e^{-0.6286(5)} \approx -406.89$$

It is easy to see that the rate of change is negative, but the magnitude is considerably less than that in part (b).

Solution Step 4:

A linear depreciation model will approximate the curve with a linear polynomial since the form is easier to use. The model comes from computing the equation of the secant line joining $(0, V(0))$ and $(10, V(10))$.

$$\text{slope} = \frac{V(10) - V(0)}{10 - 0} = \frac{15000e^{-0.6286(10)} - 15000}{10 - 0} \approx -1497.21$$

Since the model should match the value at $t = 0$, we can write a linear equation of the form

$$y - y_0 = m(t - t_0)$$

where $(t_0, t_0) = (0, 15, 000)$ and $m \approx -1497.21$. The linear model is then written as

$$y - 15000 = -1497.21(t - (0))$$

or

$$y = -1497.21t + 15000$$

Solution Step 5:

The difference between the original function and the linear model is that the rate of change of the original function is larger in magnitude early on and then is smaller than the linear model for later times. Note that the rate of change is different for different times in the original function and is a constant value of 1497.21 for the linear model. The only reason to use a linear model is that it is a much easier function to analyze.