

Problem Definition

Problem 39. Write the expression as a sum, difference, or multiple of logarithms using properties of logarithms.

$$\ln \frac{3x(x+1)}{(2x+1)^2}$$

Hint: Make sure you factor the polynomials in the expression.

Solution Step 1:

One way to proceed is to use the property relating logarithms of quotients to difference of logarithms. That is

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

This allows use to write

$$\ln \frac{3x(x+1)}{(2x+1)^2} = \ln(3x(x+1)) - \ln(2x+1)^2$$

Solution Step 2:

Next we can work on each of the two terms. Using the rule about logarithms of products

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

we can write

$$\ln(3x(x+1)) - \ln(2x+1)^2 = \ln(3) + \ln(x) + \ln(x+1) - \ln(2x+1)^2$$

Using the exponent rule

$$\log_b x^a = a \log_b(x)$$

allows the final expression to be written

$$\begin{aligned} \ln \frac{3x(x+1)}{(2x+1)^2} &= \ln(3) + \ln(x) + \ln(x+1) - \ln(2x+1)^2 \\ &= \ln(3) + \ln(x) + \ln(x+1) - 2\ln(2x+1) \end{aligned}$$