

Problem Definition

Problem 21. Find the derivative of the function.

$$y = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$$

Solution Step 1:

We can make the problem a bit easier using properties of logarithms to rewrite the function in the following way.

$$\begin{aligned} y &= \ln\left(\frac{\sqrt{4+x^2}}{x}\right) \\ &= \ln(4+x^2)^{1/2} - \ln(x) \\ &= \frac{1}{2}\ln(4+x^2) - \ln(x) \end{aligned}$$

This expression will be easier to differentiate.

Solution Step 2:

Now we will differentiate the new expression using properties of the derivative and the chain rule on the first term.

$$\begin{aligned} y' &= \frac{d}{dx}\left(\frac{1}{2}\ln(4+x^2) - \ln(x)\right) \\ &= \frac{1}{2}\frac{d}{dx}(\ln(4+x^2)) - \frac{d}{dx}(\ln(x)) \\ &= \frac{1}{2}\left(\frac{1}{4+x^2}\right)\frac{d}{dx}(4+x^2) - \frac{1}{x} \\ &= \frac{1}{2}\left(\frac{1}{4+x^2}\right)(2x) - \frac{1}{x} \\ &= \frac{x}{4+x^2} - \frac{1}{x} \\ &= \frac{x^2}{x(4+x^2)} - \frac{4+x^2}{x(4+x^2)} \\ &= \frac{x^2 - 4 - x^2}{x(4+x^2)} = -\frac{4}{x(4+x^2)} \end{aligned}$$