

Problem Definition

Problem 47. Determine the equation for a tangent line to the graph of the following function at the given point.

$$y = x \ln(x) \quad (1, 0)$$

Solution Step 1:

The first step is to compute the derivative of the function. Using the product rule we can compute the derivative as follows.

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (x \ln(x)) \\ &= \left(\frac{d}{dx} x \right) \ln(x) + x \left(\frac{d}{dx} \ln(x) \right) \\ &= (1) \ln(x) + x \left(\frac{1}{x} \right) \\ &= \ln(x) + 1 \end{aligned}$$

Solution Step 2:

With the derivative, we can compute the slope of the tangent line and use this with the given point to write the equation of the tangent line in point-slope form. That is,

$$y - y_0 = m(x - x_0)$$

with $(x_0, y_0) = (1, 0)$ and $m = y'(1) = \ln(1) + 1 = 0 + 1 = 1$. Substituting these values into the equation for the line gives

$$y - 0 = (1)(x - 1)$$

or

$$y = x - 1$$