

Problem Definition

Problem 53. Find dy/dx implicitly.

$$4x^3 + \ln(y^2) + 2y = 2x$$

Solution Step 1:

Differentiating both sides of the equation with respect to x gives

$$\frac{d}{dx} (4x^3 + \ln(y^2) + 2y) = \frac{d}{dx} (2x)$$

Applying the derivatives to the terms gives

$$\frac{d}{dx} (4x^3) + \frac{d}{dx} (\ln(y^2)) + \frac{d}{dx} (2y) = 2$$

or

$$12x^2 + \frac{d}{dx} (2 \ln(y)) + 2y' = 2$$

where the property relating the logarithm of a power and a constant multiple has been used on the second term. Finally, we can write

$$12x^2 + \frac{2}{y} y' + 2y' = 2$$

Solution Step 2:

Now we need to solve for y' to complete the solution process. The equation

$$12x^2 + \frac{2}{y} y' + 2y' = 2$$

can be rewritten as

$$\left(\frac{2}{y} + 2\right) y' = 2 - 12x^2$$

Dividing by the coefficient of y' gives

$$y' = \frac{2 - 12x^2}{\frac{2}{y} + 2} = \frac{y(1 - 6x^2)}{1 + 2y}$$

The last expression is obtained by multiplying the top and bottom of the rational expression by $1/2$ and by y .