

Problem Definition

Problem 31. **Effective Yield** The effective yield is the annual rate i that will produce the same interest per year as the nominal rate. r .

- (a) For a rate r that is compounded n times per year, show that the effective yield is

$$i = \left(1 + \frac{r}{n}\right)^n - 1$$

- (b) Find the effective yield for a nominal rate of 6% compounded monthly.

Solution Step 1:

The yield is determined by the amount produced in a year. The return on an investment with nominal rate r compounded n times is

$$P = A \left(1 + \frac{r}{n}\right)^n$$

The net amount gained in a year is given by

$$Y = A \left(1 + \frac{r}{n}\right)^n - A$$

where A is the initial amount invested. Then

$$Y = A \left(\left(1 + \frac{r}{n}\right)^n - 1 \right)$$

and the yield, i , is the coefficient multiplying A in the yield.

$$i = \left(1 + \frac{r}{n}\right)^n - 1$$

Solution Step 2:

For the nominal rate 6% compounded monthly ($n = 12$) the effective yield will be

$$i = \left(1 + \frac{0.06}{12}\right)^{12} - 1 \approx 0.0617$$

or the effective yield is approximately 6.17%.